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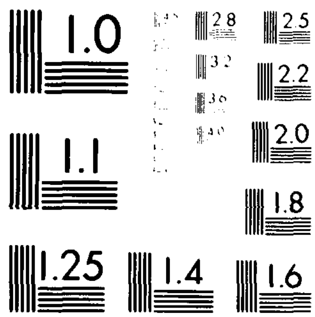

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AN ANALYSIS OF DISCOUNTING
PROCEDURES AND RISK ANALYSIS
TECHNIQUES FOR USE IN THE
DEPARTMENT OF DEFENSE

AFIT/GSM/SM/79D-16

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AN ANALYSIS OF DISCOUNTING PROCEDURES AND RISK ANALYSIS TECHNIQUES FOR USE IN THE DEPARTMENT OF DEFENSE

9 Master's THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science

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by

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Dennis D. Dailey, M.S.

Captain USAF

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Graduate Systems Management

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Preface

The study presented here is an attempt to clarify the issues in the debate over the choice of a discount rate to be used to evaluate public investments. These issues are then applied to the specific investment environment which exists within the Department of Defense.

Also presented is a methodology for evaluating the risk of a proposed investment project. This risk analysis is accomplished within the framework of the economic analysis but is considered as an issue apart from the choice of a discount rate. Hopefully this study will contribute to a better understanding of both public discounting and risk analysis.

I would like to gratefully acknowledge the support provided by Mr. Oscar Goldfarb, Deputy for Supply and Maintenance, in the Office of the Assistant Secretary of the Air Force (Research, Development and Logistics) who sponsored this study. Thoughtful comments provided by Colonel Donald Ernst, Deputy Director for Programs Evaluation, Directorate of Programs; Colonel Ronald Brence, Deputy Director, Directorate of Contracting and Acquisition Policy; and Mr. Glenn Gotz of the Rand Corporation, were also very helpful in clarifying some of the issues addressed in this study. Finally,

I would like to acknowledge the guidance and thought provoking questions provided by my thesis advisor Professor Joseph Cain.

Table of Contents

List of Figures

Figure		Page
19	Cost Distribution for New Facility Using a 10 Percent Discount Rate	96
20	Cost Distribution for Existing Facility Using a 15 Percent Discount Rate	98
21	Cost Distribution for New Facility Using a 15 Percent Discount Rate	100
22	Savings Distribution Using a 5 Percent Discount Rate	103
23	Savings Distribution Using a 10 Percent Discount Rate	1[illegible]
24	Savings Distribution Using a 15 Percent Discount Rate	105

Abstract

In this study the writer examines the debate over the choice of an appropriate discount rate for evaluating public investments. The conclusions drawn from the study are then applied to investment decisions within the Department of Defense. Two major issues are examined in detail. In the first, the social time preference discount rate is rejected, for both practical and theoretical reasons, in favor of an opportunity cost of capital rate which considers the alternative use in the private sector of the funds used in public investment projects. The second point of controversy centers on the use of a riskless discount rate versus a rate that includes the risk compensation for private investments. The riskless discount rate is rejected because it does not adequately portray the opportunity cost of public investment funds.

While a rate that includes the risk compensation present in the private sector adequately portrays the opportunity cost, it does not consider the risk inherent in the project. The writer concludes that consideration of inherent risk is a separate issue from the choice of a discount rate. One possible approach for consideration of inherent risk is presented through the use of probability distributions for cost and benefit variables in the investment analysis and

the use of Monte Carlo simulation to generate a probability distribution for the net present value of an investment. The study concludes with an application of this probabilistic approach to an economic analysis which had previously been accomplished within the Air Force Logistics Command.

AN ANALYSIS OF DISCOUNTING PROCEDURES AND RISK ANALYSIS TECHNIQUES FOR USE IN THE DEPARTMENT OF DEFENSE

I. Introduction

Background

In 1966 the Department of Defense published DODI 7041.3 which established a policy of using a ten percent discount rate for the purpose of comparing alternatives for proposed new programs or projects. The purpose of this policy, as restated in AFR 178-1, is to ensure that no DOD investment is undertaken without explicit consideration of the alternative uses of the funds which that investment absorbs or displaces. The ten percent rate is meant to represent "an estimate of the average rate of return on private investment before corporate taxes and after adjusting for inflation" (Ref 1:11). This policy seems intuitively reasonable, but when one begins to investigate the theoretical basis for this approach, a great deal of controversy comes to light. In order to gain some insight into this controversy, one must first review the rationale and procedures for the use of discounting in the private sector.

Discounting in the Private Sector

The rationale for the use of discounting is to adjust future receipts and costs that result from the investment to account for the "time value of money." This concept is based on the premise that a dollar received at some future date is worth less than a dollar received at present (Ref 14:45). In these days of inflation this may seem like a gross understatement, but it is important to recognize that this is true even if there were no inflation whatsoever.

This concept has long been accepted by private enterprise and applied to capital investment decisions through the use of a group of methods collectively referred to as discounted cash flow measures. The two most commonly used methods are the internal rate of return (IRR) approach and the present value (PV) approach.

The procedure for using both of these methods involves first estimating the cash flow for each period over the life of the investment. These estimates are then used in the following net present value formula in slightly different ways depending on the method employed.

$$NPV = -x_0 + \sum_{j=1}^{n} \frac{x_j}{(1+i)^j} \qquad (1)$$

(Ref 43:253)

where x_0 = initial cost of the project,

x_j = cash flow in period j,

n = number of periods in the life of the project (usually expressed in years),

i = discount rate for the PV approach or internal rate of return for the IRR approach (both are annual rates if the periods are in years).

The objective of the internal rate of return approach is to determine a value for i such that NPV equals zero. This value, then, is the rate of return that a particular investment is expected to produce. The present value approach takes the opposite tack by starting with a predetermined discount rate and solving for NPV (Ref 41:73-75).

The theoretically correct discount rate for use with the present value approach is the marginal cost of capital which is defined as the cost of each additional dollar of new capital raised during the current year (Ref 43:501-502). This concept is an application of the economic theory of the firm which states that it should operate at the point where marginal revenue equals marginal cost in order to maximize profit. When applied to capital budgeting, marginal revenue is taken to be the rate of return on investments, and marginal cost is the marginal cost of capital (Ref 9:446).

A rate which can be computed and used more readily is a weighted-average cost of capital. This rate is calculated by weighting the cost to the firm of debt, preferred stock, and equity capital (Ref 43:475-492). It represents a marginal cost if the weights used correspond to the actual proportions of new capital obtained from each of these sources (Ref 41:115). Thus, a project evaluated using the present value approach and a weighted-average cost of capital would be accepted if NPV were positive. Such a project would be expected to yield a rate of return in excess of the cost of the capital invested therein.

In the private sector both the IRR and PV methods are used and ordinarily lead to the same investment decisions. There are at least three situations, however, where the two approaches will have contradictory results. The first situation is one in which two or more mutually exclusive projects with different receipt streams are being considered. This situation is illustrated in Figure 1. Since the internal rate is always computed at NPV equal to zero, Project A will always be the best choice when the IRR method is used. In contrast, when the PV method is used the choice made is dependent on the discount rate used. At rates greater than i', Project A, with the higher NPV, is chosen. At rates below i', the opposite is true. Thus, if the cost of capital were less than i', the IRR decision to accept Project A would be the wrong choice (Ref 5:112-113).

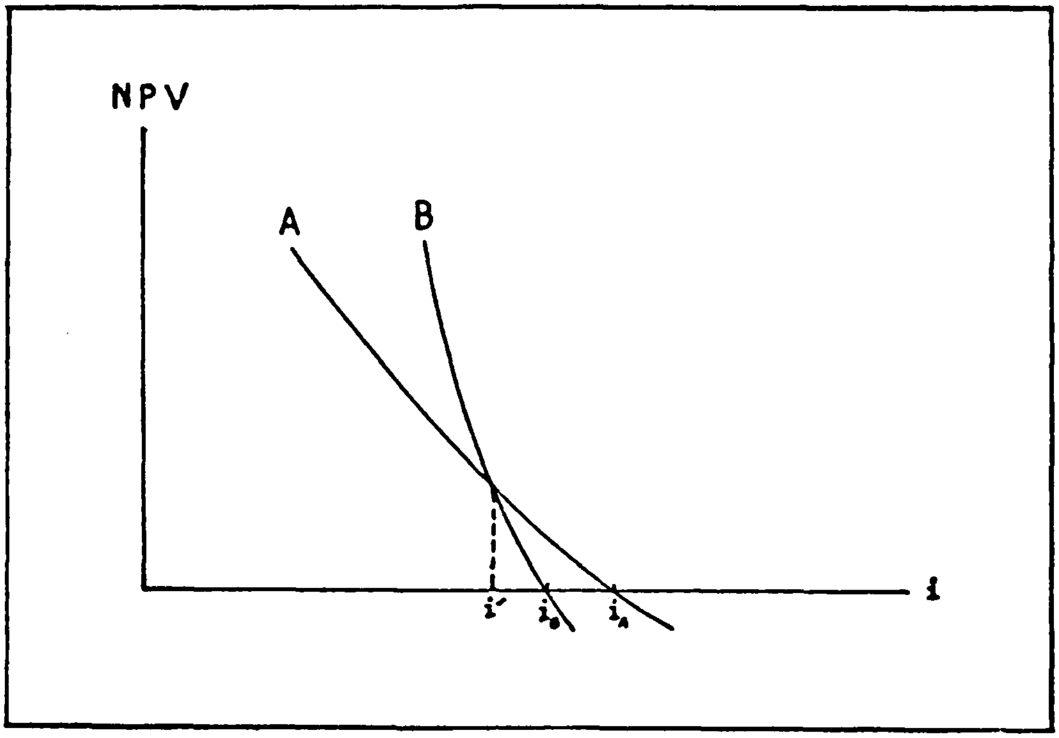

Fig 1. Comparison of IRR and PV Methods for Different Receipt Streams (Ref 5:113)

The second situation occurs when the size of projects are different. To illustrate, consider two projects with a life of one year and the following costs and cash flows.

	Project A	Project B
Initial Cost	-1000	-2000
Return after one year	1200	2320
IRR	20%	16%
NPV (discounted at 10%)	90.91	109.09

As can be seen, even though Project A has a higher IRR, its NPV is less than Project B simply because it is a smaller project. This may not be a problem if one can invest in two Project A's, but this may not always be possible (Ref 5:113-114).

The third situation is one which may occur if some future cash flow is negative as shown in the following illustration.

	Period		
	0	1	2
Cash Flow	-72,727	170,909	-100,000

(Ref 7:43-44)

Figure 2 shows that this project actually has two values for IRR, 10 percent and 25 percent. Under these circumstances it is impossible to make a decision on how a project should be ranked (Ref 5:115-116).

Any of the above problems could occur when considering investment projects in DOD. This is particularly true of the first situation since many of the investment decisions made are choices between mutually exclusive projects. Another situation encountered in DOD tends to support the use of the PV approach, that is, discounting only cost streams. This occurs in cases where benefits resulting from an investment are difficult or impossible to quantify.

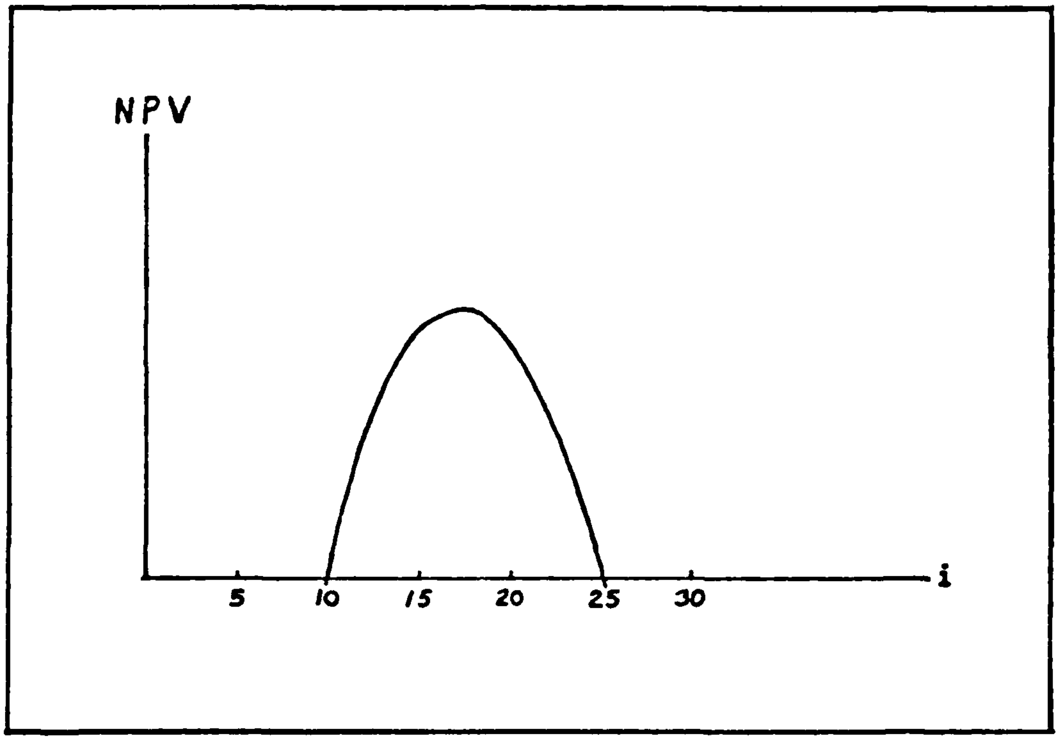

Fig 2. Multiple Internal Rates of Return

In these cases the PV approach can still be used to discount the cost stream, and a choice can be made on the basis of the lowest discounted life cycle cost.

All of the above reasons tend to support the direction in AFR 178-1 which specifies the use of the present value approach when preparing economic analysis required under DODI 7041.3. The present value approach, however, is not without complexities of its own. As stated above, the predetermined rate of discount used in the private sector is the cost of capital which is relatively easy to compute.

The main problem encountered when the discounting procedure is transferred to analysis of public investments is the choice of an appropriate rate to be used in place of the cost of capital.

Discounting in the Public Sector

The question of which discount rate is most appropriate for public investment analysis has been an issue of debate among economists for decades. Two reports, one issued in 1968 by the Congressional Subcommittee on Economy in Government (Ref 40) and another issued in 1976 by The Rand Corporation (Ref 34), summarize the main issues in this debate.

Both reports point out that, in an economy with a perfect capital market, the Government need only look at the interest rate observed in the market to determine the discount rate it should use. Because the capital market is imperfect, however, a wide range of interest rates can be observed. This fact not only makes any single observed rate inappropriate, it also contributes to the controversy surrounding the selection of an appropriate surrogate for the perfect market rate. There are two major schools of thought regarding the selection of the appropriate rate, the opportunity cost of capital approach and the social rate of time preference concept.

The premise of the opportunity cost of capital approach is that the government should seek to maximize the well being

of the nation by maximizing the national wealth. In order to accomplish this, the proponents of this concept advocate the selection of a discount rate that reflects the opportunity cost of the capital withdrawn from the private sector for public investment. The use of this rate would guarantee that no public project would be undertaken yielding a lower rate of return than could have been earned if the funds had been left in the private sector (Ref 40:12-13).

In opposition to the opportunity cost approach are those who advocate a social rate of time preference concept. This concept holds that opportunity cost does not properly account for the desire of society to provide for the well being of future generations. In order to ensure that they are adequately provided for, it is argued that the government should increase the level of public investment. This would be accomplished through the use of a low social rate of interest which would make a larger number of governmental investments feasible (Ref 40:10-11).

A third possible discount rate explored by the subcommittee is the cost to the federal government of borrowing (Ref 40:11-12). The use of this rate corresponds to the use of the cost of capital for private investments. This possition presumes that the government should act like a private firm and accept all investments with returns which exceed its cost of borrowing. Although the subcommittee rejected

this view of the government as an investor of capital, they did not condemn the use of the government borrowing rate in specific instances, as will be seen below.

Investment Decisions Under Uncertainty

The subcommittee report favored the opportunity cost of capital approach and listed several methods for estimating this rate. Each method uses realized private returns on investment in the estimation process which leads to still another point of controversy. Any investment undertaken in the private sector involves some degree of risk and uncertainty.[1] Therefore, when realized returns on private investment are used as a basis for estimating a discount rate, it will include an average allowance for the risk present in those investments (Ref 40:14) [illegible]ng this rate to evaluate public investments then implies that they have risk characteristics similar to those present in private investments. The subcommittee stated that this might be an acceptable practice but also recognized another approach.

> . . . the subcommittee finds worthy of continued study the suggestion that a basic minimum-risk interest rate be used by the federal government and that explicit allowances be made for risk and uncertainty in the benefit and cost estimates of each public investment. (Ref 40:14)

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[1]Some authors make a distinction between risk and uncertainty. These terms will be used interchangeably here, however, to mean the possibility of an actual return other than that expected.

There are numerous techniques which can be used for making explicit risk allowances in the analysis, but one of the most promising is the use of a probability distribution approach (See Refs 19, 20, 21, and 28 for examples). This approach is based on the fact that the costs and revenues which constitute the elements of cash flow in the net present value equation are random variables rather than constants. By specifying the probability distribution for each of these variables, one can account for the risk present in an investment project due to random variations in these variables.

One variation on the use of this approach is a simulation model for forming a frequency distribution of possible outcomes (Ref 19). This process involves the use of a computer program which uses random numbers to repeatedly simulate the results that might be expected for the actual investment. The data gathered from a large number of these simulations can then be used to determine an expected value, variance, and frequency distribution for NPV. This simulation approach shows particular promise as a method of making an explicit allowance for risk in economic analyses.

Objectives

After conducting a preliminary investigation of the above topics, it became evident that additional study in the areas of public discounting and risk analysis techniques

could enhance the decision making process in the Department of Defense. Accordingly, the following objectives were established to guide this thesis effort.

1. Analyze the appropriateness of the ten percent discount rate presently in use in DOD.

2. Examine the various methods for making explicit allowances for risk in the economic analysis process with emphasis on the probability distribution approach.

Research Methodology and Scope

The appropriateness of the ten percent DOD rate cannot be analyzed without a review of the entire controversy surrounding the choice of a discount rate for public investment analysis. Consequently, Chapter 2 investigates the debate between the advocates of the social time preference rate and the social opportunity cost rate while Chapter 3 examines the controversy surrounding the treatment of risk in public investment decisions. It should be noted that this analysis does not present any information that has not appeared elsewhere. It is simply a consolidation of the views of the experts in the field followed by an attempt to apply these views to the specific public investment environment which exists within the Department of Defense.

Chapter 4 examines in greater detail the use of the simulation approach to develop a frequency distribution for net present value. Also examined are the various tech-

niques available for developing probability distributions for the cost and benefit elements of cash flow.

The techniques developed in Chapter 4 are presented in an illustrative example in Chapter 5. An actual economic analysis previously conducted within DOD is used and the results compared with those obtained using the probabilistic approach of this thesis.

The final chapter presents a summary of the research effort, conclusions drawn therefrom, and recommendations for further study.

II. The Social Rate of Discount

The social discount rate is defined as the minimum rate of return that a proposed project must show before it will be accepted for public investment. In terms of the present value criteria, it is the discount rate used in Eq (1) to evaluate public projects. The question of what this rate should be has been among the most discussed and controversial issues in the area of public expenditure economics. This chapter will examine the major schools of thought concerning this issue and relate them to the ten percent rate in use in the Department of Defense. As an introduction to this subject a brief review of the economic theory of saving, investment, and interest will provide the theoretical starting point from which most of these viewpoints arise.

Theoretical Background

The theory was developed as an extension of exchange and production theory to explain the allocation of resources between time periods. The model assumes one consumptive commodity--the objects of choice being current consumption claims versus consumption claims in the future.[1] Individuals

[1]These consumption claims are dollars in our society, but the stipulation that these dollars are of constant value must be added.

as consumptive decision makers each exhibit a time preference which reflects their willingness to exchange units of current consumption for units of future consumption. This time preference is usually expressed in terms of a percentage premium demanded on future consumption claims in exchange for current ones. This percentage is referred to as the individual's marginal rate of time preference. Standard economic theory concludes that the individual maximizes his satisfaction by adjusting his consumptive claims between time periods through borrowing or lending in such a way that the marginal rate of time preference is equal to the market rate of interest (Refs 12:42 and 24:508-9).

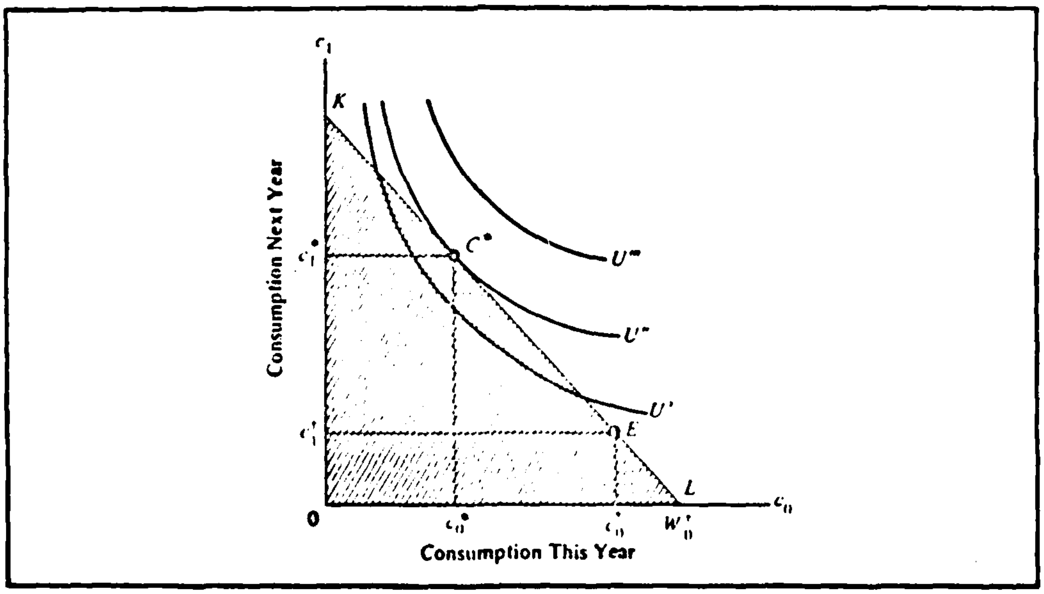

Fig 3. Intertemporal Consumptive Optimum (Ref 23:414)

An example of this analysis for two time periods only is shown in Figure 3. Point E represents this individual's endowment position with c_0^+ and c_1^+ current and future consumptive claims respectively. Line KL is called the market line and represents all of the possible trade-offs of current for future claims through borrowing or lending. Moving along the line to the northwest trades current claims in favor of future ones and represents lending. Likewise, moving southeast on the line represents borrowing. The slope of the market line indicates the market rate of interest r as follows:

$$1 + r = -\frac{dc_1}{dc_0}$$

Curves U', U'', and U''' are called indifference curves which simply represent lines of constant satisfaction for the individual. In other words, the combination of c_0 and c_1 claims corresponding to a point on one of the curves gives the individual the same satisfaction as any other point on that same curve. The slope of the indifference curves indicates the individual's rate of time preference at that particular point. Movement to the northeast to progressively higher indifference curves represents increasing satisfaction.[1]

[1]Proof of this and other propositions concerning consumer indifference curves are beyond the scope of this paper, but can be found in many microeconomics texts such as Ref 23.

Thus, it is easy to see that the point of maximum satisfaction occurs at the point of tangency between the highest indifference curve and the market line. At that point the slopes of the two lines are equal which indicates that the individual's rate of time preference is equal to the market rate of interest when satisfaction is maximized. This is indicated by point C^* in Figure 3 and corresponds to c_0^* and c_1^* of current and future consumption. Thus, this individual has maximized his satisfaction by shifting consumption to the future by saving an amount $c_0^+ - c_0^*$ thereby increasing his consumption one year hence from c_1^+ to c_1^* (Refs 12:43; 23:414-15).

One can also see from this analysis that as interest rates increase, as indicated by increased slope of the market line, the individual's marginal rate of time preference will increase to remain in equality. Up to a certain point this will also cause the individual to save more (Ref 12:43).

One other point in the figure remains to be explained, that is, the horizontal intercept of the market line shown as W_0^+. This point is called endowed wealth and is the present value at the market rate of interest of all present and future consumption claims (Ref 23:416). A person's endowed wealth limits his consumption choices over time. Thus, it can be seen that the market line is a constraint on the maximum satisfaction attainable under these conditions.

The above analysis explains how a consumer maximizes satisfaction by shifting consumption between periods through the medium of exchange. But one can also increase satisfaction though productive investment. An example is shown in Figure 4 where position E again represents the endowment position. Curve QQ is called the production possibility curve and represents the trade-offs possible in sacrificing present consumption in order to physically produce more for the future. In addition to the consumptive possibilities c_0 and c_1, q_0 and q_1 represent productive possibilities. The slope of the production possibility curve at any point is a measure of the rate of return on the marginal dollar of investment sometimes called the marginal productivity of capital. Once again, the optimum for this consumer is the point of tangency between the highest indifference curve and the production possibility curve R* (Ref 23:417-19).

Taking this process one step further will allow an analysis of the results when both production and exchange occur. This situation is depicted in Figure 5. When exchange is added to the analysis, R* is no longer optimum since a higher attained wealth can be reached by producing at a point Q*, the point of tangency between the production possibility curve and the highest attainable market line. Once this point is reached, the individual can reach his personal consumptive optimum C* through exchange along line NN.

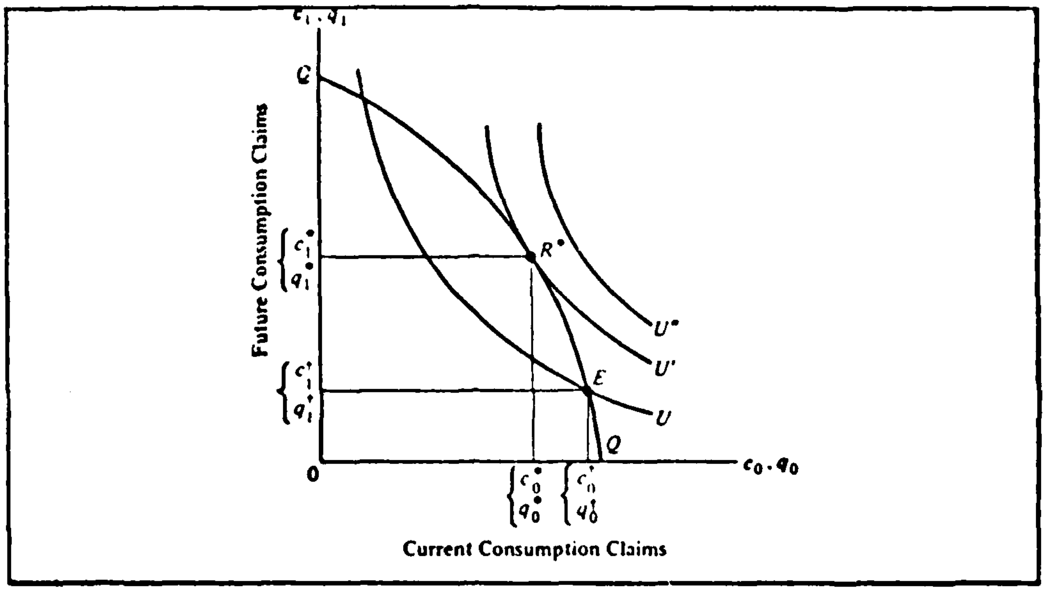

Fig 4. Intertemporal Productive-Consumptive Optimum

(Ref 23:418)

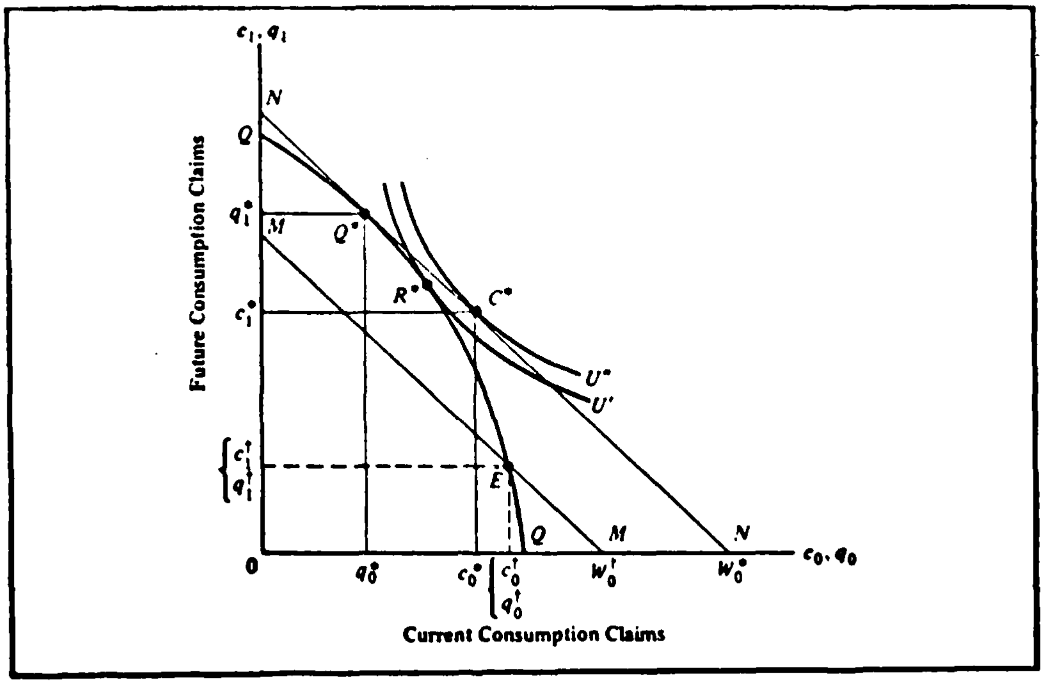

Fig 5. Intertemporal Productive-Consumptive Optimum With Exchange

(Ref 23:419)

As can be seen from the diagram, the slopes of the production possibility curve, the indifference curve, and the market line are all equal at the productive-consumptive optimum points. Thus, the marginal productivity of capital, marginal rate of time preference, and market rate of interest are all equal at the optimum solution (Ref 23:419-20).

Up to this point it has been assumed that the market rate of interest was given. In fact, the interaction of consumptive and productive decision makers in the market leads to an equilibrium rate of interest r^* which balances the supply of savings S_S and the demand for investment D_I as shown in Figure 6 (Ref 23:420-21). This being the case, r^* would be the obvious choice for the social discount rate for use in evaluating public investments (Ref 12:43-44).

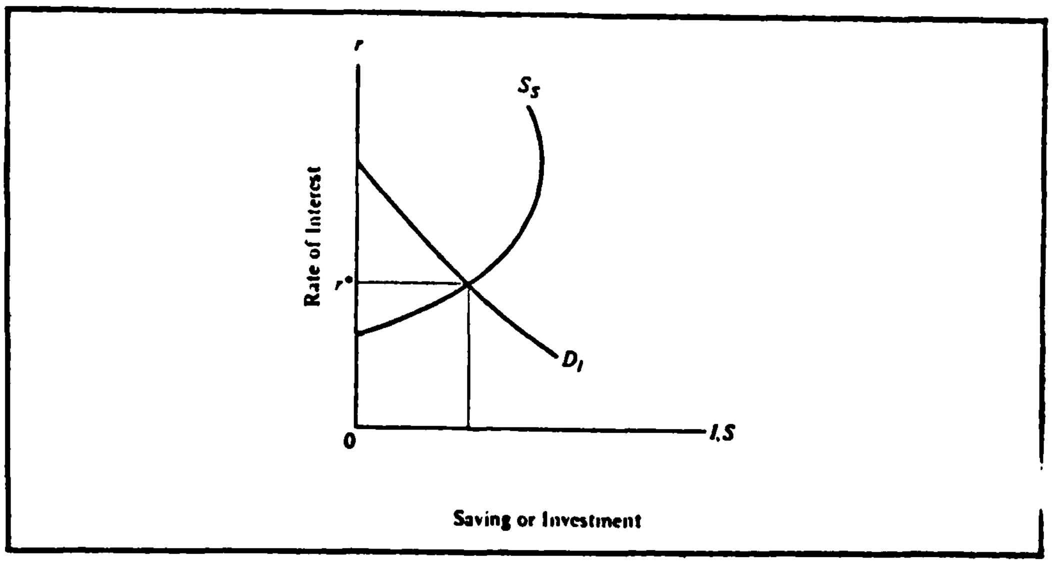

Fig 6. Saving-Investment Equilibrium

(Ref 23:420)

Of course this model is not representative of the real world situation due to the imposition of several simplifying assumptions. Among the most significant are the following:

1. The capital market is perfectly competitive, that is, consumers and producers can both borrow and lend as much as they want at the market rate of interest; and

2. That perfect foresight exists in that the rate of return on every investment is known with certainty (Ref 12:44-45).

When one returns from the world of theoretical capi[illegible] markets, it is readily apparent that there is no market equilibrium rate of interest. In fact there are a wide variety of interest rates prevalent in the market. This fact is basically caused by violations of the perfect market assumptions. The market is imperfect in that the amount of credit available is limited, borrowing rates are generally higher than lending rates for a given individual, and rates are affected by government monetary and fiscal policy. In addition, rates vary significantly according to the degree of risk inherent in the investment undertaken.

The individual time preference rate could be approximated by an average after tax rate of return on savi[illegible] while the productivity of investment could also be estimated by the average rate of return on private investment. But these rates differ--the return on investment being higher

than the return on savings. Views on which rate is relevant vary quite drastically. There are some who assert that no observed market rate is relevant, others that one or the other should be used exclusively, and still others who contend that a weighted average of the two should be used. Each of these views will be examined in turn.

The Social Rate of Time Preference

Those who ascribe to the social rate of time preference approach argue that observed market rates are irrelevant to the public investment decision. Even a perfect market rate, if one existed, would not properly reflect society's time preference of present for future consumption. The various justifications for this approach are summarized in an article by Marglin (Ref 29).

The first point of view that he discusses holds that the individuals who interact to produce the equilibrium rate of interest base their time preferences on the fact that their lives are brief and uncertain. This causes them to view future benefits as less valuable than they actually are and to shift consumption more toward the present. This in turn causes the equilibrium interest rate to be higher than it would if future generations were properly represented in the market. It is government's responsibility, according to this view, to guard the interests of these future generations who are not represented. Marglin calls this the

"authoritarian" position because of the rejection by government of individuals' private time preferences (Ref 29:96-98).

Another view which Marglin calls the "schizophrenic" argument holds that individuals actually have two time preference maps, one that governs his market actions and one that governs his choices in the political arena (Ref 29:98). Thus, it is not a matter of government rejection of the private time preference, but rather that different preferences are exhibited when considering public investments to benefit future generations.

The final argument to justify a separate social rate and the one advocated by Marglin is one that he calls "interdependence" (Ref 29:99-109). This viewpoint assumes that individuals do receive some satisfaction from increased consumption by future generations. For an individual acting alone, however, this increase in satisfaction does not offset the corresponding loss which results from his financing an investment the benefits of which he will never enjoy. When the investment is made collectively, however, the conbined investment provides an increase in the consumption by future generations which is large enough to offset this loss. But there is no way for an individual to impose his preference for this collective investment on other individuals through the market mechanism. Only through government action can compliance by everyone be ensured. The fact that these types of collective

investments do not exist in the market is the primary reason that the market rate of interest is rejected as irrelevant to public investment decisions.

Feldstein adds another reason for the inappropriateness of the perfect market rate (Ref 15:364). His argument is based on the use of social rather than private productivity of capital. He contends that social productivity may be higher than private productivity of capital. The reasoning for this claim is based on the fact that private return on investment is calculated net of payments to factors of production while, for society as a whole, these payments represent increased factor incomes and are actually a gain rather than a cost.

Feldstein also provides an explanation of the means by which government can provide "proper" representation for future generations in investment decisions (Ref 15:369-76). To do this he uses the indifference curve analysis shown earlier in the chapter, but he extends it from analysis of individual decisions to analysis of society's decisions. The situation as he depicts it is shown in Figure 7. Curve AB represents the social productivity of investment for society as a whole. Since it is a measure of social productivity, the slope at any point along this curve is the social opportunity cost of funds diverted from private to public investment.

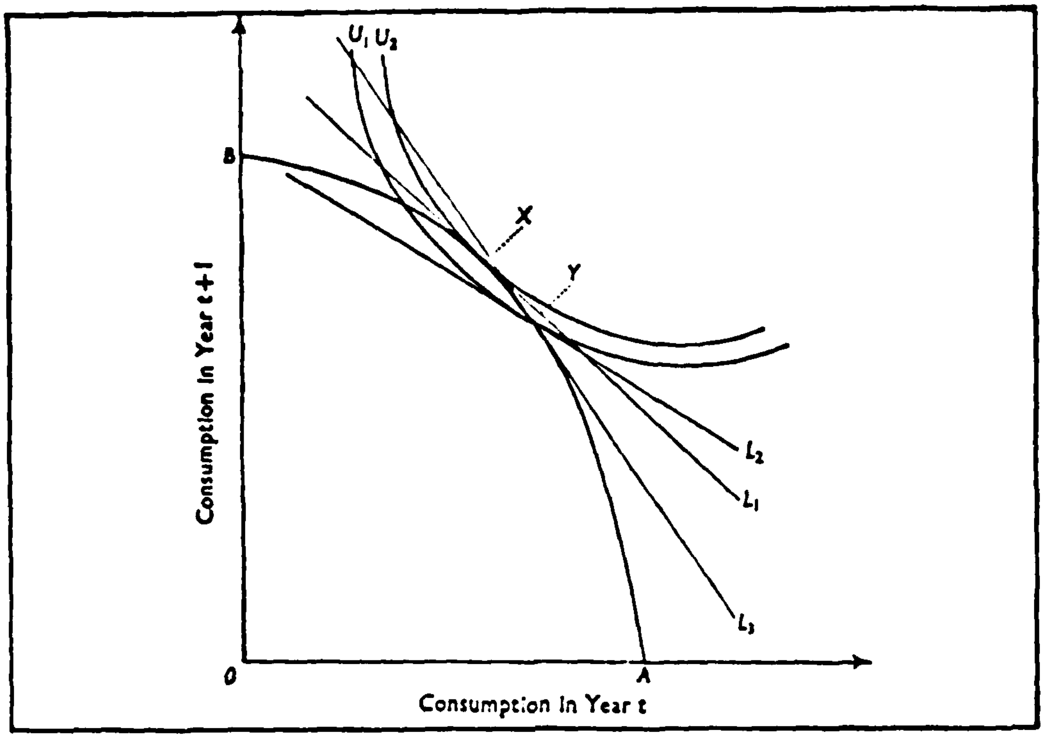

Fig 7. Indifference Curve Analysis for Public Investment (Ref 15:374)

The indifference curves U_1 and U_2 represent social time preference functions. The slope of these functions at any point thus indicates society's rate of time preference. It should be noted that there is no market line in this analysis. Feldstein explains that this is because society as a whole cannot redistribute the benefits from public investment by borrowing or lending (Ref 15:369). This limits the analysis to a "Robinson Crusoe" situation where investment is the only means of redistributing consumption over time.

In this analysis the optimum consumptive position would be point X, the point of tangency between the social investment productivity curve and the highest social indifference curve. The corresponding social rate of time preference would be derived from the slope of Line L_1. But this rate would presumably be lower than the market rate as explained earlier. Thus, the only way that this position would ever be attained is if the total levels of both public <u>and</u> private investment were determined as a matter of public policy (Ref 15:374-5). This would be accomplished through a government cheap money policy which would lower interest rates in the market so that they are the same as the social time preference rate. This would encourage longer term investments in both public and private sectors (Ref 6:796). But the government may not be able to use this kind of monetary policy for both political and technical reasons. The productivity of private investment would then exceed the social time preference rate as shown by point Y in Figure 7. In this situation the government can only apply the social time preference rate to its own investment decisions.

At point Y the slope of the social investment productivity curve, depicted as Line L_3, is the opportunity cost of funds transferred to public investments. Line L_2 depicts the slope of indifference curve U_1 at the same point and is the social time preference rate which the government should use to evaluate public projects. It is interesting to note

that this social time preference rate used for the "second best" solution at point Y is less than both the opportunity cost and the social time preference rate that would have been used at the theoretical optimum point.

This analysis provides a theoretical explanation for why a lower discount rate should be used, but it cannot be used to actually compute a social time preference rate. This is true for at least two reasons. First, actually determining society's preference function is impossible (Ref 15:371). Second, the fact that the time preference rate is dependent on the position on the investment productivity curve while at the same time this position is dependent upon the amount of investment, and thus ultimately the time preference rate itself, makes it impossible to determine the rate in advance of the investment analysis. Despite these problems, Feldstein advocates a determination of the social time preference rate through the use of subjective estimates (Ref 15:376). He also believes that these estimates should be made by democratic administrators within the government. He bases this belief on the premise that this administrative decision would meet the requirements of democracy as long as overall government actions were acceptable to the electorate.

Other views on how this discount rate should be chosen include the "authoritarian" view that a "more rational"

government time preference should replace those of individuals (Ref 15:366). This "more rational" rate would presumably be determined by "rational" public servants. Marglin's "interdependence" view, on the other hand, recognizes that individuals actually exhibit a social time preference and that government should account for the interests of future generations only to the extend of these time preferences. The actual determination of these preferences would be done subjectively by the elected officials as representatives of the people (Ref 15:366-67).

Before proceeding to a discussion of the problems with the social time preference approach, it must be pointed out that the use of this approach versus the use of the opportunity cost of capital approach is not a completely dichotomous situation. The proponents of the former approach do not reject the use of opportunity cost to evaluate public projects. What they do object to is incorporating these costs into the discount rate. They assert that because of the presence of many interest rates in the market, the source of investment funds would have to be identified in order to properly represent opportunity cost in the discount rate. A better way to allow for opportunity cost, they contend, is to place a "shadow price" on the funds used in a project (Ref 15:361-62). Shadow pricing, as the term is used here, is simply an increase in the cost of these resources

for the purpose of analysis to account for better alternatives foregone (Ref 31:82-3). This method is advocated by both Marglin and Feldstein. Eckstein takes a different approach by applying a cutoff cost-benefit ratio greater than unity to account for opportunity cost (Ref 13:53), but the effect is exactly the same as that obtained by shadow pricing.

Problems with the social time preference approach are of both a theoretical and a practical nature. On the practical side, the subjective nature of the estimates for the social time preference rate constitute one of the major problems with its use. Not only must the rate be chosen by subjective estimate but so must an appropriate shadow price or cutoff cost-benefit ratio to account for opportunity cost. This problem is addressed by Haveman who holds that the presumption that the legislative body of government can deal with these choices assumes that politicians can discipline themselves more than has been shown to date. He further states that "to require of elected representatives such a double jeopardy decision is to grant by default ultimate determination of this issue to those interests and those politicians who gain from both low discount rates and a low cutoff benefit-cost ratio" (Ref 18:948-49). Eckstein points out that the two step process of choosing a discount rate and a cutoff cost-benefit ratio seems unworkable for the

same reasons. If this is true, he claims, then the interest rate itself must incorporate the opportunity cost of capital (Ref 13:55).

The same sort of argument could be made for an administratively determined rate. Most administrators are anxious to adopt new programs, thus, the choice of a social time preference discount rate becomes an exercise in promoting programs rather than providing for the well being of future generations. As long as there is no sound empirical basis for deriving the rate of social time preference and its choice is left to subjective judgment, this type of problem will be evident.

In addition to these practical problems, there are also theoretical shortcomings with this approach. The best argument is expressed by Tullock (Ref 38). He reasons that, given the past rate of per capita economic growth, future generations are going to be wealthier than those at present. Therefore, increasing government investment to provide for future generations actually taxes the poor to help the rich. As Tullock remarks, "It is possible to imagine an individual making this sort of sacrifice, but it is hard to imagine his imposing it on his contemporaries for charitable reasons" (Ref 38:334-5).

The argument is also made that a lower social time preference rate should be used for the protection of such

things as natural resources which might be used up or destroyed. This is not a valid reason for lowering the discount rate used throughout government. A more appropriate approach would be to approve these types of investment on a case by case basis without regard to what the rate of return will be (Refs 6:801 and 27:143).

Given these theoretical arguments plus the aforementioned difficulties of actually determining a rate to be used and a way to incorporate opportunity costs, most economists have rejected this approach in favor of the opportunity cost of capital concept.

The Social Opportunity Cost of Capital

This concept assumes that government investment in public projects displaces an equal amount of funds from the private sector. The basic premise of the approach, therefore, is that the social discount rate used to evaluate government projects should be chosen in such a way that a project will be accepted only if its benefits exceed the value of the alternative use of those funds if they had been left in the private sector. Only in this way, it is contended, can government guarantee that higher valued private uses are not displaced by lower valued public ones (Refs 6:789-90 and 18:949).

The concept relies upon observed market rates to provide a measure of opportunity cost. As noted earlier,

opinions vary as to which market rates should be used. These differing opinions result primarily from the assumption made concerning which private sector uses are actually displaced; consumption, investment, or both. Other variations occur according to the assumed source of the investment funds. At least four variants of the basic opportunity cost position are evident in the literature depending on which assumptions are made.

The first of these positions assumes that public investment displaces only private investment. Estimates of the opportunity cost rate, therefore, should use a measure of the productivity of private investment. One measure which has been used for this purpose is a weighted average of the rates of return observed from businesses in various corporate sectors and the noncorporate sector (Ref 37). The weights used in the averaging process were based on the relative flow of capital and equipment investment from each of the sectors considered.

In a second position, Baumol adopts the view of most economists by assuming that public investment reduces both private investment and consumption (Ref 6). The empirical estimation process that he uses, however, is very similar to the one used in the first position. He justifies this approach by arguing that consumers actually indicate how they feel about their foregone consumption through the rate

of return they are providing to business firms through their purchases of goods and services. Hence one need look no further than these rates to estimate the opportunity cost of public investment (Ref 6:792).

The final two views also assume that public investment reduces both consumption and private investment but take different views as to how the public investments are financed. One assumes that they are funded primarily through taxation while the other assumes government borrowing as the source of these funds.

Harberger addresses the case where government borrowing is assumed as the source of funds (Ref 17). He argues that increased public demand for credit is the instrument by which private investment and consumption are displaced. The increased demand causes interest rates to rise. The higher rates cause corporate and noncorporate sectors of the private economy to invest less as well as causing individuals to consume less (as a result of increased saving). In order to measure this cost one could trace the restrictive effect of the government borrowing on various sectors of the capital market and compute a weighted average according to how much each sector was affected (Ref 17:58). In defending the use of the opportunity cost of borrowed funds, Harberger cites the fact that there is a definable pattern in which government borrowing displaces private investment. This pattern

provides a basis for the weights used to compute the weighted average opportunity cost. Changes in the way funds are raised through taxation, on the other hand, have no such standard pattern as to their effect on each sector of the private economy. For this practical reason he prefers the opportunity cost of borrowed funds to that computed assuming taxation (Ref 17:65).

Those supporting a rate computed based on taxation contend that it is the proper rate if for no other reason than the fact that, in actual practice, public investments are financed by taxation and not borrowing (Ref 13:55). The result of taxes is to reduce private spending for both investment and consumption. For this reason one must consider both rates of return of private investment and interest rates reflecting private time preference when computing the opportunity cost rate of discount. Estimation of this rate requires that a specific set of tax changes be assumed in order to identify what private expenditures are foregone. One can then measure the returns from those alternative uses and weight each according to the relative effect of the tax on consumption or investment in that particular area (Ref 18:950). In a 1969 article, Haveman supported this last position and contrasted the various views and the assumptions made in each as shown in Table I (Ref 18).

Table I

Summary of Differences Between Opportunity Cost Concepts

Location of Private Sector Displacement	Means of Displacement	Interest Rates by Which to Value Displacement	Relative Weights Used to Compute Average Return Foregone
1. Business Investment Exclusively	Physical extraction of resources from private sector by public sector	Observed average rates of return before taxes on capital invested in private business	Percentage breakdown of business investment in plant and equipment
2. Consumption and Investment	Physical extraction of resources from private sector by public sector	Observed average rates of return before taxes on capital invested in private business	Percentage breakdown of business investment in plant and equipment
3. Consumption and Investment	General tightening of the capital market due to government borrowing	Interest rate on government bonds plus adjustments for taxes	Relative distribution of private spending cutbacks due to the increase in government borrowing
4. Consumption and Investment	Diversion of private sector spending caused by taxes	Effective rates on which consumers make saving-borrowing decisions--Effective rates of return on private investment	Relative incidence of a particular tax or pattern of taxes on consumption and investment sectors and their subsectors

(Ref 18:951)

Disagreement on which of the assumptions shown in the table are correct is one of the problems with this approach since these assumptions determine, in part, the value obtained for the social discount rate. But even if a consensus of opinion were obtained on one of these views, there are still computational difficulties to overcome when estimating the rate. The most prevalent is the calculation of the rate of return in each sector. This computation requires additional assumptions concerning the stock of capital in each sector and the effect of various taxes. In addition, one must decide which years will be included in the data (Ref 34:6). The results obtained will again depend upon these additional assumptions. Since there has been little agreement in the past on which assumptions are appropriate, this will probably continue to be a problem in future attempts to estimate this rate. But regardless of the differences in assumptions, various attempts to actually compute a discount rate using views three and four in the table have all resulted in a value in the 7-10 percent range (Refs 13:56, 17:63, and 39:45).

The rate adopted by DOD adheres to the opportunity cost of capital approach. More specifically, it assumes that public investment funds are drawn from both corporations and consumers (Ref 1:5). Although the 10 percent rate was not empirically derived, it still falls within the 7-10 percent

range determined by more rigorous methods. Thus, at least with respect to opportunity cost, the DOD rate conforms to the theoretical approach accepted ty the majority of the economic profession and is of an appropriate magnitude. In addition, the constancy of the DOD rate seems to be justified since the real opportunity cost of capital is historically very stable (Ref 34:27).

One final point can be made concerning the use of the opportunity cost rate. Since observed market rates are used to compute this rate, it will reflect some allowance for the risk present in private investment. This is viewed by some as a problem of overstatement of risk when a project is undertaken in the public sector. This view and others concerning the treatment of risk will be examined in the next chapter.

III. The Effects of Risk and Uncertainty

In the perfect capital market model of the previous chapter, one will recall that uncertainty was assumed not to exist. Perfect foresight of the returns on investment eliminated any risk to the investor. In reality, a wide range of risky investments exist from riskless government bonds to extremely risky ventures such as oil drilling. Risk can be more explicitly defined in terms of the variability of possible outcomes from an investment. Figure 8 shows the probability distribution of returns for two investments with the same expected return but different variability. Project A has a higher probability of achieving the expected return and a narrow range of possible outcomes.

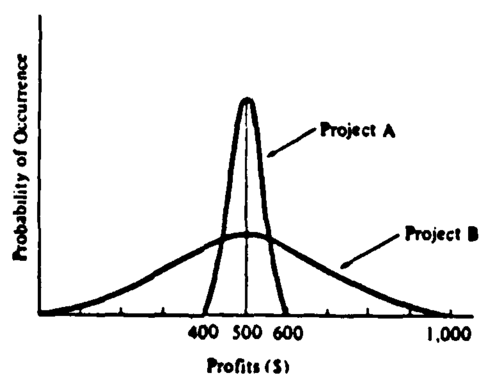

Fig 8. Variability as a Measure of Risk (Ref 9:66)

Project B, on the other hand, has a lower probability of achieving the expected return and a much wider range of possible outcomes. Project B is thus viewed as being riskier than Project A.

The measure of risk that will be used here is the coefficient of variation (ν) defined as the standard deviation of the distribution divided by the expected value. This coefficient measures the variability of the distribution and also compensates for the size of the investment.

Because of the existence of risk in investments, a discussion of the concept of risk aversion is necessary. In theory, one can identify three attitudes toward risk: a desire for risk, an aversion to risk, and an indifference toward risk. Brigham and Pappas state that "both logic and observation suggest that business managers and stockholders are predominantly risk averters" and, given a choice between two investments as depicted in Figure 8, would choose Project A with the lower risk (Ref 9:68). This phenomenon can be explained in terms of the diminishing marginal utility of money, that is, each additional increment of income has less utility than the last. Again viewing the two projects in Figure 8, Project B has a possibility of a much larger return than Project A but at the same time it also has the possibility of a much lower return. An individual exhibiting a diminishing marginal utility for money will derive more

dissatisfaction from the lower return of Project B than satisfaction from the higher return. Thus, this individual would choose Project A in order to decrease the possibility of a lower than expected return even at the expense of decreasing the possibility of a much higher return. This phenomenon can be observed in government as well as private business managers. The negative effects of a large cost overrun on a government project are much greater than the positive effects of a large cost savings. Thus, government managers are also likely to be risk averters.

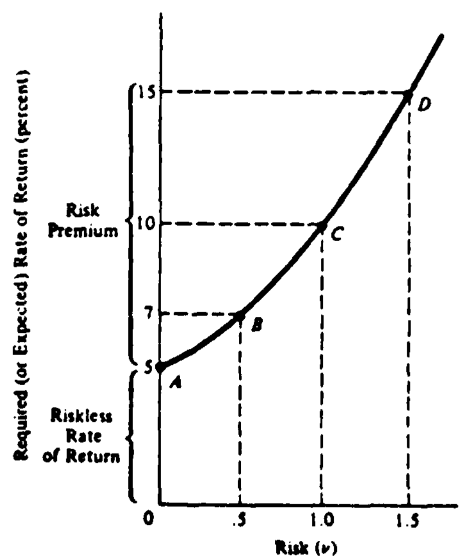

Fig 9. The Relationship Between Return and Risk (Fig 9:75)

The net results of risk aversion is that the value of investments are discounted below their expected value depend ing on the degree of risk involved. A more convenient way

to view this concept is shown in Figure 9. The figure is a graphical depiction of a hypothetical risk function for a risk averse investor. As can be seen, this investor demands a risk premium over and above the riskless rate which increases with the increased risk of an investment. The risk premium is, in effect, a cost incurred as a result of the additional risk.

When applying discounting techniques in the public sector, a point of controversy arises over the question of whether risk compensation should be a social as well as a private cost. Those who hold that it is not a social cost contend that a riskless rate should be used to evaluate public projects. The opposing view holds that the rates of return which include a risk premium should be used to properly allocate resources between the public and private sectors. This chapter explores the rationale for each of these views followed by the development of the method of allowing for risk that is recommended in this thesis.

Use of a Riskless Rate

A review of the literature on the subject reveals at least two major justifications for the use of a riskless rate of discount to evaluate public projects. Both views contend that the cost of risk aversion is socially irrelevant but for different reasons. The first view is expressed by Samuelson and Vickery (Refs 33 and 42) and is based on a

diversification argument. The contention is that since government invests in a large number of diverse projects, the law of large numbers would indicate that the expected return of all of the projects taken together is virtually certain. Thus, government acts as "a device for mutual reinsurance" of possible low returns on projects by correspondingly higher returns on others. This result would dictate the use of a riskless rate for public project evaluation.

The second view is sometimes included with the first but actually constitutes a different viewpoint as expressed by Arrow and Lind (Ref 3). The argument in this case is that spreading of risk over all the members of a large population reduces the risk to each to zero in the limit. Thus, it is this spreading risk rather than diversification which justifies the use of a riskless discount rate (Ref 3:366).

There are two implicit assumptions in both of these viewpoints which lead to their rejection. First, it is assumed that the returns from public investments are either independent of other components of national income or that the average covariance among the rates of return is zero (Ref 34:11). Second, it is assumed that there are no insurance markets for much of the risk present in the private sector, and therefore, government can insure against this risk more effectively than can the private market mechanism.

With regard to the first assumption, Arrow contends that even if some government investments are interdependent, they can be evaluated as a package and that even after these groupings are made, there are a large number of independent projects (Ref 3:373). A counter argument is that the returns are positively correlated through the business cycle (Ref 34:11 and 36:400). Arrow replies that if stabilization policies are assumed to be successful, then this problem does not arise (Ref 3:373). Again, this is countered by the observation that "monetary and fiscal policy are not perfect instruments of national income management." Thus, stabilization policies cannot be assumed to be successful (Ref 34:11). Further, even if project diversification were possible it might not be desirable since combining investments that pay off under different economic circumstances may reduce the value of the total investment package (Ref 22:272-73).

Turning to the second assumption, it has been asserted that private insurance markets do in fact exist--namely, stock markets. Although imperfect, they do allow individual investors to diversify at a negligible marginal cost and are no less imperfect than government's ability to allocate risk bearing according to individual preferences (Ref 34:11). Some economists have even advocated the opposite extreme--that public projects be discounted at a higher rate than private projects because of this misallocation of risk bearing (Ref 36:400).

Perhaps the most convincing argument in opposition to the use of a riskless rate, however, has to do with the proper representation of opportunity cost. Arrow accepts the opportunity cost concept but uses individuals' valuation of private risky investment to establish this cost. He states that individuals discount the value of risky investments by an amount equal to the cost of risk bearing. Thus the true opportunity cost of the private invesments foregone is the value placed on them by individuals--namely, the riskless rate of return (Ref 3:374-75).

Arrow's argument is effectively countered by Baumol (Ref 6) who views opportunity cost not from the viewpoint of individual valuation of investments but rather the value of the investment to society as a whole. This view is based on the observations that society benefits from all investment projects--public and private, and that private investments taken as a whole are equally as riskless to society as the aggregate of public investment. This being the case, private investment should also be valued by society at their expected returns. Since this value includes a compensation for risk, any anticipated transfer of resources from the private sector must include the risk premium in the opportunity cost (Ref 6:795-96).

Apparent confusion concerning this role of private risk premiums as an opportunity cost led to a recommendation

of use of a riskless rate by both Hoffman and Lynn in the 1968 subcommittee hearings on interest rate policy (Ref 39).

Hoffman agrees that government programs are not riskless but contends that explicit consideration of risk by adjusting the expected costs and benefits would lead to better public investment decisions (Ref 25:27-28). Similarly, Lynn stated that

> In the absence of fully satisfactory answers to questions about the appropriate way to handle risk in government investments, the best way to proceed is to adopt (a riskless rate) as the basic discount rate. . . and to insure that each project valuation includes an analysis of the uncertainties associated with costs and benefits. (Ref 27:144)

These views were reflected in the subsequent subcommittee report as noted in Chapter 1 (pg. 10).

This recommendation is based on the assumption that explicit allowance for the risk inherent in a particular project obviates the need to consider the risk premium included in returns from private investments foregone. This assumption ignores the role of private risk premiums as opportunity costs. The use of explicit allowances for risk may still be useful in decision making, as will be discussed more fully below, but the opportunity cost of displaced private investment must still be properly represented.

The writer is of the opinion that the arguments presented in this section favor of the use of a "risky" discount

rate are more persuasive than any that have been used to support the use of a riskless rate. The choice of which "risky" rate should be used is the only question which remains to be answered. This question is explored in the next section.

The Choice of a "Risky" Rate

Various recommendations have been made as to how the "risky" rates of return observed in the market should be used to establish a discount rate for government use. The variations arise mainly due to the distinction between the risk factor which should properly be included in opportunity cost and the risk associated with a particular project under consideration (referred to here as "inherent risk"). The nature of this distinction can be seen by considering the implications of the use of a procedure that allows for only the risk associated with opportunity cost.

A discount rate derived using a weighted average of private investment and consumption foregone, as shown in methods three and four from Table I in the last chapter (pg. 35), includes an allowance for the average risk in private investment. As shown in the previous section, this risk is included for the purpose of properly representing opportunity cost. But what would happen if this rate were used to evaluate two alternative proposals with risk characteristics like Projects A and B in Figure 8 (pg. 38) without allowing

for the differences in inherent risk between those projects? Since the expected returns are the same, this kind of analysis would lead to indifference as to which project to choose. But most decision makers, being risk averse, would not be indifferent and would choose Project A as being the most desirable. Thus, it is essential that the inherent risk of each project be considered in addition to allowing for risk in the opportunity cost.

Of course inherent project risk is never completely ignored. Many methods have been devised to allow for this risk. Some of the most common methods along with their relative merits and disadvantages are discussed below.

Allowances for Inherent Risk

Risk Premiums. One method often suggested for allowing for differences in risk between projects is the use of a risk premium. This method is based on the relationship shown in Figure 9 (pg. 40) in which the rate of return demanded increases as risk increases. Thus, if one project is deemed to have a higher degree of risk than another, a higher discount rate would be applied to compensate for the additional risk.

There are several reasons why this method is inappropriate for use in DOD. First, it presupposes that one knows what the comparative risk is for the various alternative projects. Presumably this knowledge would be gained through

some sort of subjective comparison of the factors involved. But without some measurement of the risk involved, there can be no basis for the objective assignment of a risk premium.

Even if one assumes that an appropriate risk premium can in some way be assigned, there is still doubt as to whether its use if justified. As Hoffman pointed out in the subcommittee hearings on the subject (Ref 25:28), there is no reason to believe that risk is a compound function of time which is implied if a compensation is made through the discount rate. Additionally, there is no basis for expecting similar degrees of risk for both costs and benefits.

Another problem which applies to many of the analyses done in DOD is the effect of a risk premium on a life cycle cost stream. Figure 10 shows the relationship which exists between the discount rate and both net present value (NPV) and life cycle cost (LCC). As can be observed, NPV decreases as the discount rate increases until finally it is no longer positive. Since the project with the highest NPV will be chosen, adding a risk premium to the discount rate would be appropriate in that case. Increasing the discount rate also causes LCC to decrease, but since the project with the lowest LCC will be chosen, the effect is just the opposite. Adding a risk premium in this case could make the risker project appear more attractive (Ref 25:28).

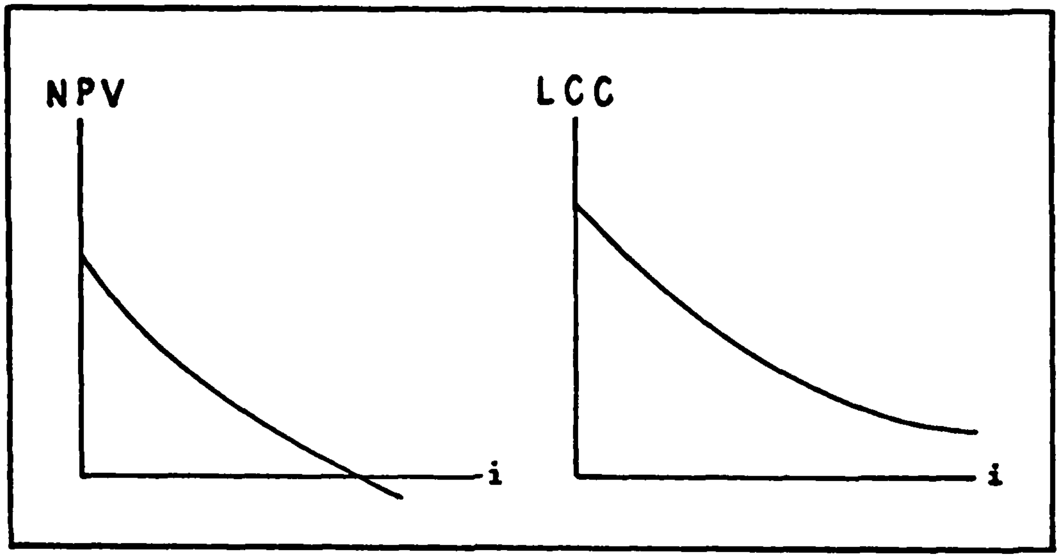

Fig 10. The Relationship Between NPV and LCC and the Discount Rate

A variation on this approach is suggested by Hirshleifer and Shapiro (Ref 24). In applying the time-state preference approach to the choice of a discount rate, these authors argue that since the variety of interest rates present in the market are, at least in part, a result of the variations in risk among private investment projects, the discount rate used for public projects should be the expected rate of return from projects in a similar "risk class" in the private sector (Ref 24:525). This approach in effect incorporates the inherent risk of a project into the discount rate to reflect the opportunity cost of displaced "similarly risky" private investments. If alternative proposals involve varying degrees of risk, the alternatives would be evaluated

using a "project-specific" rate. Since this rate could vary between alternatives, the effect is the same as the use of risk premiums with the same disadvantages as described above.

The biggest problem that would be encountered if one attempted to implement this approach is in determining the comparable risk class to which a public project should be assigned. This is particularly true in DOD where most projects have no private counterpart as noted by Somers (Ref 35). Consequently, no private rate of return would be available for use as a discount rate.

But does this situation mean that the use of private rates of return are irrelevant for evaluating a unique project as Somers suggests? This writer believes not. Somers' argument assumes that this type of public undertaking which is impractical to pursue in the private sector makes the free flow of resources between the sectors infeasible. Hence the market does not give the relevant discount rate (Ref 35:574). This argument, however, overlooks a very important fact concerning the choice among alternative project proposals. One alternative may call for a large investment now which will reduce costs in the future while another proposal may call for a smaller present investment at the expense of higher future costs. This difference in the pattern of cost streams must be taken into account through the use of the appropriate opportunity cost in the private sector (Ref 27: 140). Therefore, even if a unique project has no corre-

sponding risk equivalent in the private sector which can be used as a guide for inherent risk adjustments, there is still the opportunity cost which must be accounted for.

Because of the numerous disadvantages of incorporating an allowance for inherent risk in the discount rate, this approach is rejected in favor of allowances incorporated into the benefit and cost variables as discussed below.

Sensitivity Analysis. The method presently recommended for assessing risk in DOD projects is sensitivity analysis (Ref 1:3). This method involves testing how the final results change when key cost and benefit inputs are allowed to vary in the analysis. If the results change very little, the analyst or decision maker can be assured that errors in estimating the value of that variable will have little effect on the results of the project. If, on the other hand, the results vary significantly, then errors in the estimation of that input variable are more critical. Using this information to assess the riskiness of a project is largely subjective and depends on such things as the number of critical variables in the analysis and the degree of sensitivity of the results to changes in each.

Although this method is better than no consideration at all for risk, it suffers from several weaknesses. First, sensitivity analysis is generally accomplished by changing one variable at a time. This procedure gives the decision

maker some idea of how the results are affected by changes in the particular variable under consideration but gives him very little information concerning the possible results of changes in several variables simultaneously. Second, only changes in "key" variables are considered instead of all variables. This procedure could result in the oversight of significant changes in the total project as a result of relatively minor changes in numerous "insignificant" variables. Lastly, it provides no formal statistical measure of uncertainty and, therefore, no probability statements. This last problem makes it difficult to compare the relative risk of alternative proposals.

Direct Cost-Benefit Adjustments. One method that can be used to compare the inherent risk between alternatives was suggested by Hoffman in the 1968 subcommittee hearings (Ref 25:28). He recommended making a direct adjustment to cost and benefit variables in the analysis to reflect differences in risk. Costs would be increased or benefits decreased according to the degree of risk present.

While this approach is a step in the right direction, it has some shortcomings that make in unacceptable. As in the use of risk premiums, the assumption is made that an assessment of the risk present in a particular alternative has already been made and can be used to decide how much the costs or benefits should be adjusted. Even if a rela-

tive risk assessment is assumed to exist, there is still a problem of determining what the adjustment should be. For example, assume that one alternative has been determined to be twice as risky as another. Does this mean that the adjustment should be twice as large? This would imply that a straight line relationship exists between inherent risk and the corresponding adjustment. This kind of relationship may or may not be true. In fact, there is no practical way to determine what this relationship should be. Thus, any adjustments made will be unavoidably arbitrary. To avoid this problem while still dealing with the cost and benefit variables, a more formalized approach is required.

The Simple Triple-Value Method. The next step up in the formalization of a risk allowance procedure is a simple triple-value approach (Ref 31:372-73). In this method an upper and lower limit for each cost and benefit input variable is subjectively estimated which places a bound on the single valued estimate used previously. These three estimates for each variable are then used in the net present value equation to determine a most likely, a pessimistic, and an optimistic result. The most likely result is determined as usual by computing the net present value using the most likely estimates for each variable. The pessimistic result is computed by sutstituting the upper limit for cost variables and the lower limit for benefit variables in the net present value equation while the opposite limits are used in computing the optimistic outcome.

The pessimistic and optimistic figures for net present value determine the absolute range over which the actual outcome of the investment might fall. The range is a measure of variability and, as such, is also a measure of risk. Thus, the range in conjunction with the most likely value could be used to compare the risk between alternatives. But the only probabilistic statement which can be made using this analysis is that 100 percent of the possible outcomes are included in the range. If one wishes to determine the probability of the actual outcome falling in a narrower range, it is impossible to do using this approach. In order to make this kind of probability statement a probability distribution for the possible outcomes must be developed.

The Probability Distribution Approach

As noted in Chapter 1, the probability distribution approach recognizes that each cost and benefit element in an economic analysis is actually a random variable with some probability distribution. In order to demonstrate the use of this approach, the problem will be simplified somewhat by assuming that there is only one random variable in each time period representing both costs and benefits. Each random variable has an expected value (mean), $E(x_j)$, and variance, $V(x_j)$. Net present value, as defined in equation (1) from Chapter 1, is a function of a linear com-

bination of random variables and, therefore, is also a random variable with an expected value and variance given by equations (2) and (3).

$$E(NPV) = -E(x_0) + \sum_{j=1}^{n} \frac{E(x_j)}{(1+i)^j} \qquad (2)$$

$$V(NPV) = V(x_0) + \sum_{j=1}^{n} \frac{V(x_j)}{(1+i)^{2j}} + 2 \sum_{j<k}\sum \frac{Cov(x_j, x_k)}{(1+i)^{(j+k)}} \qquad (3)$$

$Cov(x_j, x_k)$ is the covariance between variables x_j and x_k and the double sum is over all pairs (j,k) with $j<k$ (Ref 30:185). Since the expected values for the cash flows in equation (2) are identical to the point estimates in equation (1), the numerical results achieved using these two equations will be exactly the same. Equation (2), however, recognizes that NPV is a random variable and locates the center of its probability distribution. Variance, as defined in equation (3), is a measure of the variability of NPV about the mean. The covariance term in equation (3) is a measure of the linear dependence of variables x_j and x_k.

Hillier, in his treatment of this subject, simplifies equation (3) by assuming that all of the variables are either completely independent or perfectly correlated. This causes all of the covariance terms to be either zero or one in equation (3) thus making it unnecessary to deal with covari-

ance between variables (Ref 20:447-8). Use of these assumptions may yield satisfactory results in specific cases, but in general, they are not representative of real world situations. For correct treatment, the joint probability distribution should be specified for each combination of random variables and the covariance determined by

$$\text{Cov}(x_j, x_k) = E(x_j x_k) - E(x_j)E(x_k). \qquad \text{(Ref 30:182)}$$

The complexity of this task can be more fully appreciated if one drops the previous assumption of only one random variable in each time period. There may, in fact, be a very large number of cost and benefit variables in each period. The task of determining a joint distribution for each pair of variables in both the same time period and all other time periods would probably require much more time and manpower than a more precise treatment would justify. An alternative to this monumental task is simply to estimate the covariance between variables to form the covariance matrix, but, as Hillier suggests, it is unrealistic to expect analysts to develop reliable covariance estimates (Ref 20:449). Due to this complexity, Hillier's simplifying assumption may be required of necessity.

Assuming the covariance terms have been determined and the variance computed, the next step is to place a confidence

limit on NPV. In the most general case, the cost and benefit variables could be described by any one of a number of different probability distributions; thus, no assumptions can be made regarding the distribution for NPV. In this case one must rely on Tchebysheff's Theorem to specify a confidence limit. From the theorem, the probability that the actual NPV will be within two standard deviations of the mean is .75 and within three standard deviations, .89 (Ref 30:104). This inability to assume any sort of theoretical distribution for NPV and thus decrease the confidence limit is another of the drawbacks of this analytical approach.

Another drawback is the fact that more complex problems often lead to mathematically intractable situations as noted by Graves in his attempt to apply a similar approach to the Air Force Logistics Command Logistic Support Cost Model (Ref 16:74-76). Additionally, information regarding higher moments, such as skewness, which might be of value to the decision maker are even more complex than the variance computation.

A procedure which has been applied to the probability distribution approach to overcome some of these drawbacks is a simulation model as noted in Chapter 1. The addition of the frequency distribution which simulation provides will make the information concerning such things as skewness available to the decision maker. This alone would probably

be valuable enough to justify the simulation approach, but, in addition, goodness of fit tests could be performed to determine if the frequency distribution is approximated by a theoretical distribution. Even if this cannot be done, less sophisticated techniques such as the use of a planimeter can be used to estimate the area under a given portion of the curve defined by the frequency distribution. In either case, this information can be used to estimated a confidence limit which would be a significant improvement over that which can be obtained using Tchebysheff's Theorem. The problem of dependence between variables can also be handled using this procedure by specifying in the program any functional relationships which exist.

Techniques for determining both the probability distributions for each variable and the frequency distribution using the simulation approach are explored more fully in the next chapter.

IV. A Probabilistic Approach

This thesis effort began with a look at the issue of the choice of an appropriate discount rate for use in investment decisions. The primary reason that a probabilistic approach is presented at this time is to point out that inherent cost uncertainty of a project may outweigh the conceptual differences that exist in the choice of an appropriate discount rate (Ref 25:28).

Cost overruns have been the source of a great deal of consternation within the Department of Defense. Most of the efforts to solve this problem have been in attempts to increase the accuracy of the cost estimates. There is, of course, a great deal of room for improvement in this area, but it must be recognized that there will always remain an element of uncertainty in the estimates. This reality was expressed in the report to the President and the Secretary of Defense by the 1970 Blue Ribbon Defense Panel.

> Cost estimating for development programs has apparently been too widely credited in the DOD, in industry, in the Congress, and by the public with a potential for accurate prediction which is belied by the inherent technical uncertainties in development. The precise problems which may be encountered in the process of attempting to convert into practical, producible application cannot be foreseen with accuracy. . . . the use of precontractual cost estimates as a firm baseline for measuring performance throughout the life of the

system, and the shock reaction which is forthcoming when cost overruns or growths are experienced all evidence an unwarranted degree of confidence in cost estimates (Ref 8:83).

Thus, the problem of uncertainty in cost estimates cannot be solved entirely by improving the quality of the estimates. Improvements in this area must be coupled with efforts to increase awareness on the part of decision makers concerning the uncertain nature of the estimates. One way of enhancing this awareness is to depict the cost of a project in terms of a probability distribution rather than a point estimate. In so doing, a cost interval can be specified and an explicit statement attached defining the probability that the cost will fall in that range. With cost estimates expressed in this fashion, it is less likely that "unwarranted confidence" will be placed in any specific point estimate. The means of developing a probability distribution for project cost is the subject of this chapter.

Cost Versus Schedule and Performance Uncertainty

Risk has been discussed up to this point solely in terms of cost. In fact, at least two other types of risk are present in most projects--that associated with schedule and performance (Ref 26:18). Both usually apply only to the development of a project. Schedule risk refers to completion of the project development on schedule while performance risk involves completing the project to meet certain performance

standards. Variance from either time or performance goals will affect the cost of a project. Within limits, tradeoffs exist between these two factors and the project cost.

Schedule and performance uncertainty are certainly important aspects in the development of projects; however, the analysis presented here takes a cost viewpoint. It is assumed that uncertainty present in time schedules and performance requirements and capabilities can be expressed in terms of the development cost of the project. This development cost is just one aspect of this analysis. Results using such methods as network analysis to develop a probability distribution for cost to undertake a project (Refs 2 and 4) would be used as only one of the input variables in the analysis. Additional probability distributions are also developed for the variables associated with the cost of operation in the out years and the benefits, if applicable, which accrue over the life of the project. A discussion of how a probability distribution can be specified for each of these variables will now be undertaken.

Developing Probability Distributions

Before one can develop probabilistic estimates for cost and benefit variables these variables must be identified. This phase of a risk analysis is extremely important because overlooking some of the relevant variables may result in a poor assessment of the dominant risk of the project (Ref 4:13).

A detailed description of this phase will not be undertaken here, but suffice it to say that all available information on the project should be reviewed in an attempt to identify the relevant variables.

Assuming that the variable identification phase is complete, the next step is to consolidate all of the information gathered concerning these variables. When dealing with quantitative data, this consolidation will entail developing a representative probability distribution for each variable of interest. This is not to say that qualitative aspects of the project are unimportant. On the contrary, they are often extremely important, but they must be dealt with by considering the consequences of these potential problem areas through means other than quantitative evaluation.

There are numerous techniques which can be used to develop probability distributions. The particular technique employed depends in part on the amount of objective data available which can be used to make the estimates.

Subjective Probability. Early in the planning phase of a project when an economic analysis is prepared, it is very likely that few data will be available from which to estimate cost and benefit variables. In this kind of situation the use of subjective probability techniques provide the only alternative available for quantifying the uncertainty in these variables. According to the subjective probability

concept, the probability of an event is the degree of confidence placed in the occurrence of the event by an individual based on the evidence available to him (Ref 4:17). The individual providing the subjective probability should have expert knowledge about the variable under consideration in order to ensure the validity of the estimate. The evidence may include both objective and subjective data, but objective data are included only to the extent that the expert incorporates them into his subjective judgment.

There are various methods which can be used to elicit subjective probability distributions from knowledgeable individuals. Atzinger, et al (Ref 4:24-64), illustrates several of the methods which can be used to generate discrete distributions. There discussion includes choice between gambles, standard lottery, modified Delphi, and modified Churchman-Ackoff techniques. A brief look at these methods will provide some insight into the procedural steps used.

In the choice between gambles technique, the expert is first given a choice between two possible gambles: a hypothetical 50-50 chance of winning a certain amount of money, or the real world gamble of winning the same amount of money if the actual outcome of a cost[1] variable falls in a

[1]The variables are referred to here as cost variables; however, it should be understood that these techniques are just as applicable to benefit variables or, for that matter, time or performance variables.

given range. If the expert prefers the real world gamble, this implies that he subjectively assigns a probability greater than .5 to that outcome. If, on the other hand, he prefers the hypothetical gamble, then his subjective probability of that outcome occurring must be less than .5. Next, the chances of winning the hypothetical gamble are either increased or decreased, depending on whether the subjective probability was greater than or less than .5, and the same question asked again. This process is repeated in an iterative manner until the expert is indifferent as to which gamble to choose. At that point, the last value used for the chance of winning the hypothetical gamble is taken as his subjective estimate of the probability of the cost variable falling in that range. The next higher of lower range is then used and the same procedure is repeated to obtain a probability for that range. This process is continued until the probabilities for all possible ranges have been established. The final result is a discrete probability distribution similar to that shown below.

Cost Interval	<$4000	$4000-$5000	$5000-$6000	$6000-$7000	>$7000
Probability	.0	.2	.6	.2	.0

The standard lottery technique uses a procedure which is very similar but instead of giving the expert a choice between gambles, he is given a choice between the chance of winning a given amount of money if the actual outcome of a cost variable falls in a given range or holding 50 out of 100 tickets (as an example) on a lottery which pays the winner a like amount. The number of tickets <u>held</u> is varied, keeping the total number available constant, until the expert is indifferent as to which choice to make. The proportion of tickets held at that point is then taken as his subjective probability of the cost actually falling in that range. As with the choice between gambles technique, this process is repeated until a complete probability distribution is generated. It may seem that these two methods go to more trouble than is necessary by relating the real world probabilities to betting situations, but it has generally been determined that these betting situations will elicit better responses than will direct questioning concerning the probabilities (Ref 4:24-25).

The modified Delphi technique is used when several experts are available from which subjective probability estimates can be obtained. This technique involves eliciting probability responses from each member of the group individually, usually through a questionnaire, along with the reasons for assigning the probabilities to each possible value. A feedback procedure is then used in which the

responses of other group members are made available to each individual. With this new information available to them, the individuals are again asked to provide probability estimates. Repeating this process generally results in a consensus, or very close to it, after only a few iterations. Any differences remaining are resolved by averaging the responses.

The modified Churchman-Ackoff technique differs from those discussed above in that it does not use betting situations to elicit probability estimates but uses instead a comparative approach between possible values for the variable. Specifically, the expert is asked if one possible value of the variable is greater than, equal to, or less than other possible values and by how much. The details of this method are too involved to repeat here, but readers interested in more information on this or any of the other methods discussed above can refer to reference 4 for a detailed discussion.

The major drawback of all of the above methods is the significant amount of time required to accomplish the procedure. A somewhat less time consuming approach is to obtain a lowest possible estimate, a most likely estimate, and a highest possible estimate for each variable. The variable is then assumed to be beta-distributed with the following probability density function:

$$f(x) = K(x - L)^{\alpha}(H - x)^{\beta} \quad \text{for } L \leq x \leq H \qquad \text{(Ref 11:13)}$$

where L = lowest possible value

H = highest possible value

α, β = beta parameters

The most likely value is used as the mode (m) of the beta function which can be defined in terms of the beta parameters as

$$m = \frac{\alpha}{\alpha+\beta} \qquad \text{(Ref 11:15)}$$

This approach provides a continuous probability distribution with finite end points, unimodality, and a shape which can be varied by simply changing the values of the beta parameters. It has been used for both time distributions, as in PERT (Ref 10:546-48), and cost distributions (Ref 11). The ability to change the shape of the distribution allows the analyst to specify a distribution which is skewed either left or right depending on the relationship between the most likely and the high and low estimates and also to specify the variance depending on the relative difference between the high and low estimates. But this ability to change the shape of the distribution is also a drawback since there are

literally an infinite number of different distributions which can be specified. Dienemann avoided this problem in his analysis by specifying nine different combinations of beta parameters as representative of most of the possible distributions of cost (Ref 11:12-15). The nine distributions that result are shown in Figure 11 to provide some feeling for how changes in the parameters affect the shape of the distribution.

Another possible alternative is the use of a triangular distribution as depicted in Figure 12. This distribution retains the desirable aspects of the beta distribution, that is, finite end points, unimodality, and the ability to change the shape and variance, while avoiding the problem of specifying beta parameters. The distribution has the following density function:

$$f(x) = \frac{2(x-L)}{bc} \qquad \text{for } L \le x \le M$$

and

$$f(x) = \frac{2(H-x)}{c(c-b)} \qquad \text{for } M \le x \le H$$

where

$$b = M - L$$

$$c = H - L$$

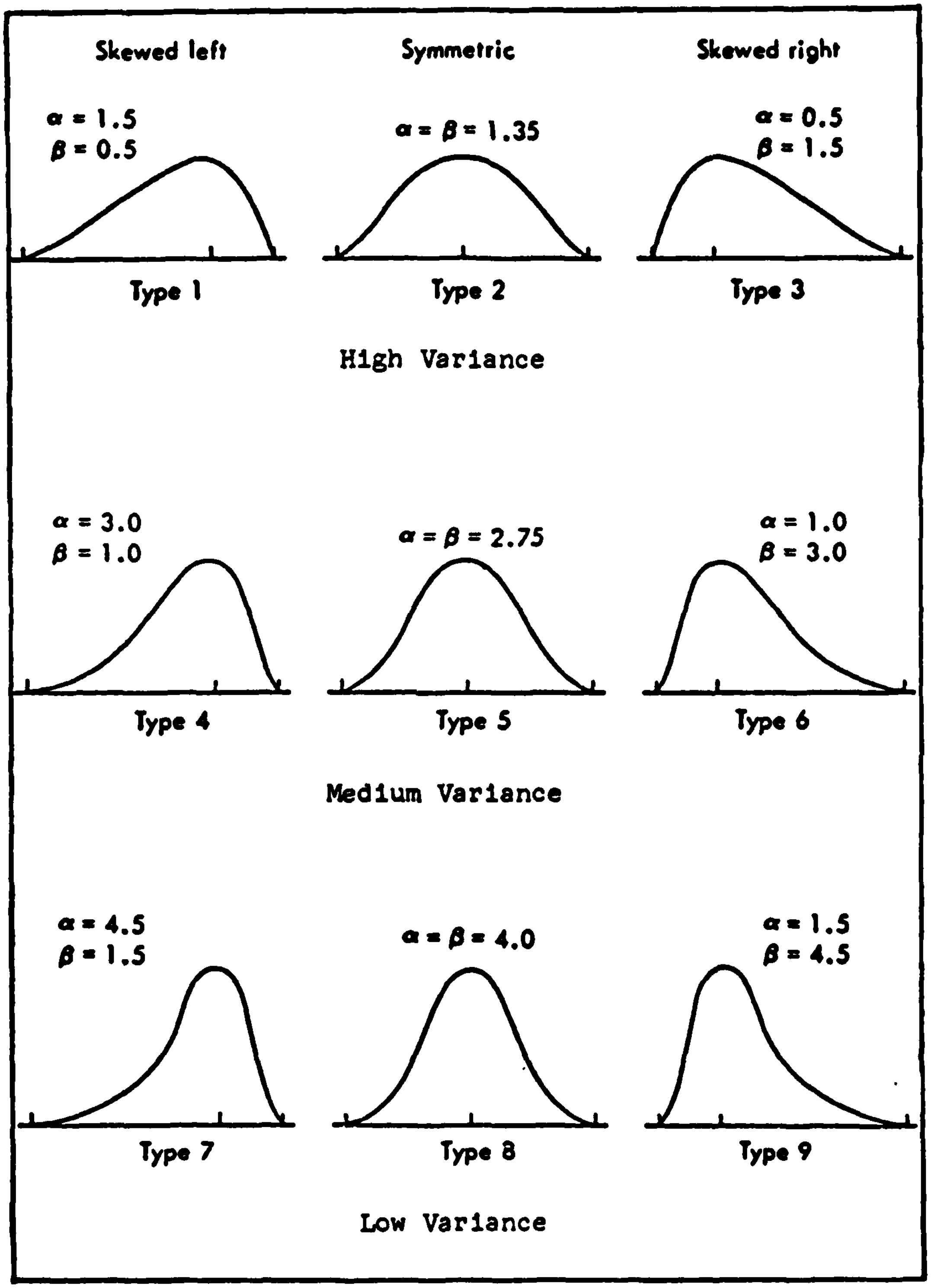

Fig 11. Representative Examples of the Beta Distribution

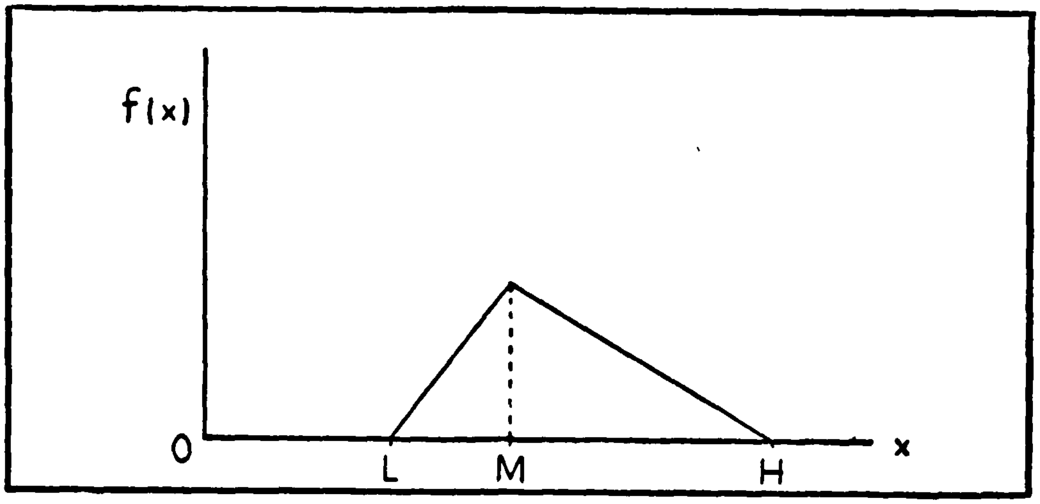

Fig 12. The Triangular Distribution

and L, M, and H are the lowest possible, most likely, and highest possible values, respectively. The function must be given in two parts due to the discontinuity at the modal (most likely) value.

Although this distribution seems quite simple, studies by the World Bank have revealed that estimates obtained using this distribution come remarkably close to actual results (Ref 32:13). Additionally, this same staff study viewed the beta distribution as weighting the value assigned to the most likely estimate too heavily. This is illustrated by the fact that the mean of the beta distribution is computed by weighting the modal value four times more heavily than the end points (Ref 10:547). Computation of the mean for the triangular distribution, on the other hand, weights

the modal value equally with the end points thus eliminating this bias (Ref 2:82).

Empirical Distributions. If historical or test data are available, it is possible to generate more objective probability distributions. The simplest way to quantify the data is by frequency of occurrence in the form of a discrete distribution. An example of this kind of distribution is shown below where historical data on man hours to complete a certain task have been grouped into one hour intervals.

Man Hours Required	Frequency of Occurrence	Probability of Occurrence	Cumulative Probability
5-6	5	.125	.125
6-7	9	.225	.35
7-8	14	.35	.7
8-9	10	.25	.95
9-10	2	.05	1.000
	40	1.000	

From the table it can be seen that between five and six hours were required on five occasions, six to seven hours on nine occasions, and so forth. The probability of the time required being in any given range is then computed by dividing the frequency for that range by the total number of data points.

Although this type of distribution is easy to develop, it also has some disadvantages. First, with relatively few

data points available, the empirical distribution may not properly represent the actual distribution. In the example above, for instance, there may be some probability of a value less than five or greater than ten. Second, a discrete distribution as used above may not be appropriate for some variables which are continuous in nature. These problems can sometimes be remedied by using a theoretical distribution.

Theoretical Distributions. Both discrete and continuous probability distributions can be represented by theoretical functions with specific mathematical form. These theoretical functions can be useful for several reasons. They simplify the calculation of probabilities and also provide a specific mathematical function with which to "fit" the empirical data. Additionally, even if empirical data are not available in a specific situation, past experience with similar situations in which a theoretical distribution accurately represented the probabilities may make it reasonable to adopt the same distribution in the present situation. The discussion presented here will be confined to the distributions which tend to properly represent certain types of data. The reader interested in the specific mathematical form for these functions can refer to an introductory statistics text such as reference 30. Figure 13 presents a representative shape for each of the distributions discussed below.

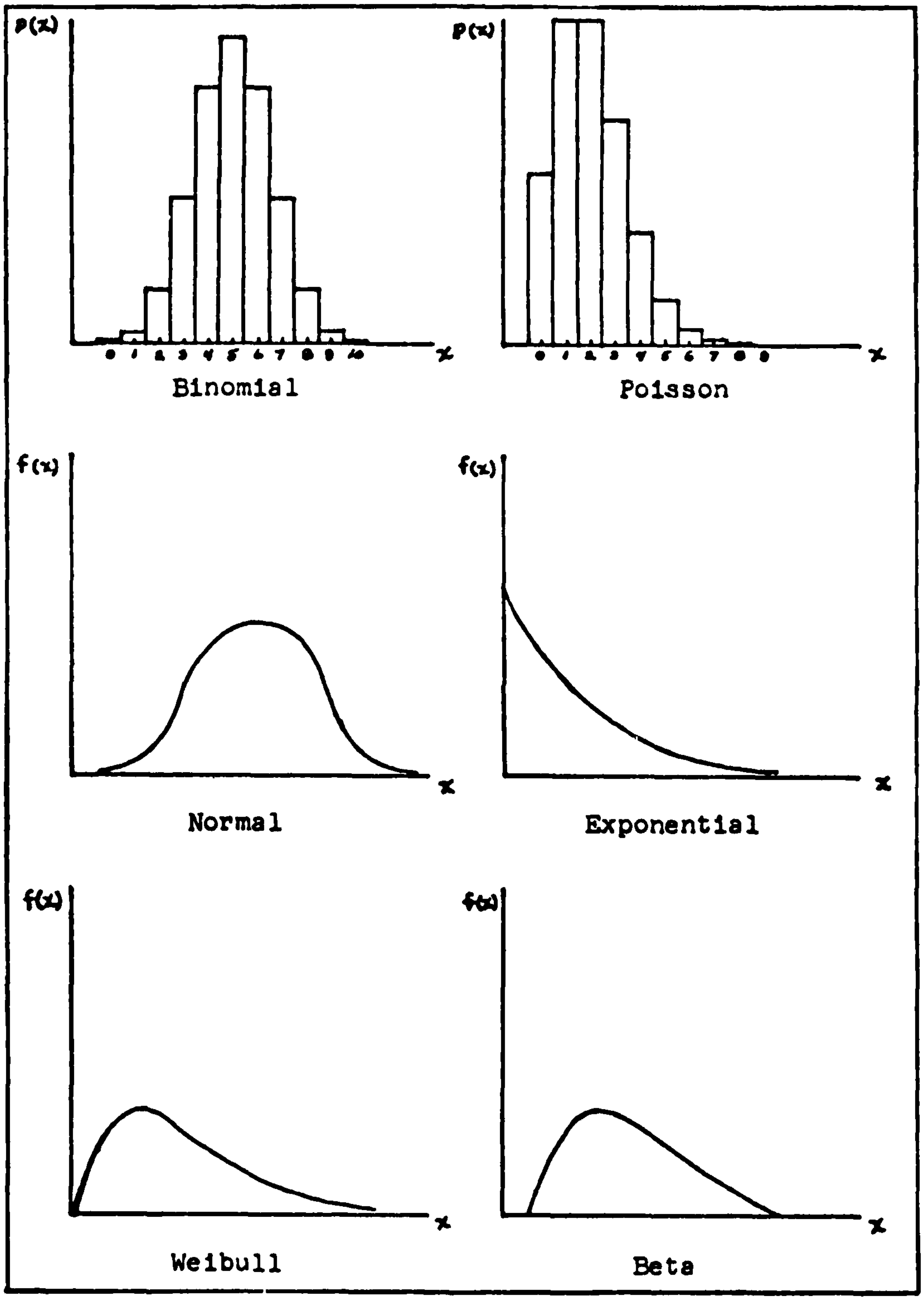

Fig 13. Representative Theoretical Distributions

The binomial and Poisson distributions are often useful in representing the probability of occurrence of discrete events where the event can be only one of two possible outcomes. Although somewhat limited due to their discrete nature, they can be useful in estimating such things as the demand for a certain product or service. The Poisson distribution is particularly useful in reliability applications to estimate the number of failures of an electronic system per unit time. An example of the use of the binomial can be seen in Graves' use of this distribution to represent the fraction of units repairable on station (Ref 16:73).

Some of the continuous theoretical distributions which are useful include the normal, exponential, Weibull, and beta distributions. The merits and uses of the beta distribution have already been discussed in the previous section and will not be covered again. The normal is the common bell shaped distribution which describes a variety of real world phenomena. An example of this can be seen in a production situation. If a machine is set-up to produce ball bearings of diameter one inch, the distribution of actual diameters produced can be described by the normal distribution with a mean of one inch. Because of its pervasive nature in describing such phenomena, the normal curve commands a central role in sampling theory.

The exponential distribution is most commonly used to represent the time to failure of electronic equipment when the failure rate is constant over time. When the failure rate cannot be assumed constant such as during the "burn-in" phase or wearout phase of the equipment, a Weibull distribution can be used which will account for the time dependent nature of the failure rate. Both of these distributions are used extensively in reliability and life cycle cost models.

This short discussion of subjective, empirical, and theoretical probability distributions is by no means exhaustive but is intended to provide the reader with some idea of how variables can be represented stochastically in an economic analysis. Only after this has been achieved can a meaningful evaluation of risk be accomplished.

As discussed previsouly, Monte Carlo simulation is one possible approach which can be used in the evaluation phase. A brief discussion of this process will now be presented.

Monte Carlo Simulation

The Monte Carlo technique is a procedure for reconstructing probability distributions based on the generation of uniformly distributed random numbers (Ref 10:491-497). An example of how this is accomplished will serve to clarify the procedure.

Suppose that the estimate for a cost variable has a triangular distribution with a low value of $100, a most

likely value of \$150, and a high value of \$200. The cumulative distribution function (CDF) for this variable is plotted in Figure 14. If one were to choose a random value for x, the probability that the cost will be less than or equal to that value can be determined graphically by locating the entering argument on the x-axis, projecting vertically to the cumulative curve, and then horizontally to locate the probability as shown in Figure 14. Mathematically, this is equivalent to using a value for x in the CDF to determine the probability. If one were to equate the CDF to another random variable u as follows,

$$F(x) = u, \tag{4}$$

then a random value for x would result in a random variate for u. If this process is repeated again and again, the resulting random variates for u are uniformly distributed in the range (0,1).

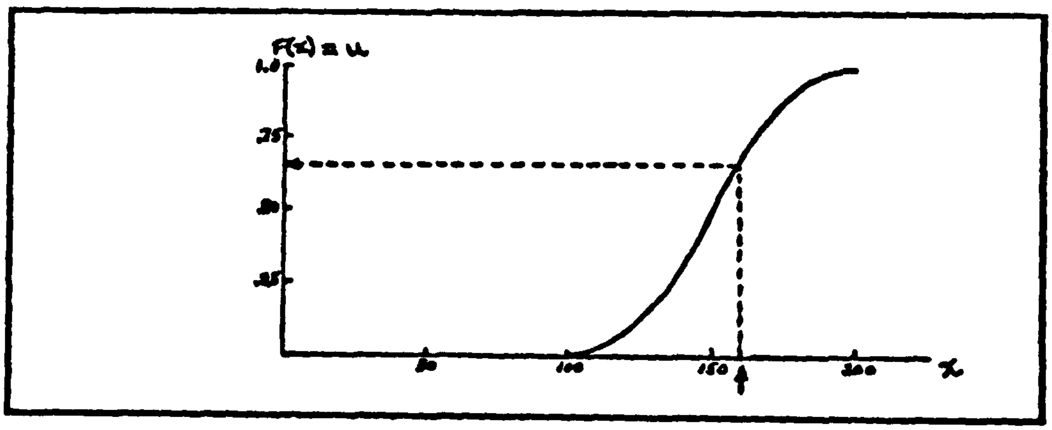

Fig 14. CDF for a Triangular Distribution

In order to reconstruct probability distributions, the Monte Carlo technique reverses this procedure by using random values for u. Graphically, this reverses the path shown in Figure 14. From the entering argument for u one would project horizontally to the curve and then vertically to determine a random variate for x. Mathematically, this is equivalent to the inverse of Equation (4) or

$$x = F^{-1}(u) \tag{5}$$

By repeatedly generating values for u in Equation (5) and plotting the resulting random variates for x in a relative frequency histogram, a close approximation of the original distribution results. Thus, all that is required to reconstruct a distribution is a convenient method for generating a large number of uniformly distributed random decimals. This can be done by using a random number table, or more commonly, a computer routine designed to generate pseudorandom numbers.

In application this technique is not used to reconstruct probability distributions for which a probability density function (pdf) is already known. Instead, it is used to construct distributions for random variables for which the pdf is unknown. In a situation where a random variable is a complex function of other random variables, it may not be possible to determine the pdf for the new variable. But the Monte Carlo technique can be used to determine random variates

for each of the variables that make up the functional relationship which in turn can be used to determine random variates for the new variable. Once again an example will clarify this procedure.

Assume that labor cost for a year is determined as follows:

$$Y = \frac{x_1 x_2}{x_3} \tag{6}$$

where Y = labor cost,

x_1 = standard labor hours,

x_2 = cost per labor hour,

x_3 = plant effectiveness factor

Further assume that estimates for x_1, x_2, and x_3 are triangularly distributed with the following low, most likely, and high values.

	L	M	H
x_1	15,000	20,000	35,000
x_2	$8.50	$10.00	$11.50
x_3	.90	.95	.97

By using the Monte Carlo technique to determine random variates for x_1, x_2, and x_3, a random variate for Y can be computed using Equation (6). Using a computer routine to repeat this "experiment" a large number of times makes it possible to determine many random samples for Y using very little computer time. As before, the random samples can then be plotted in a relative frequency histogram which will closely approximate the actual distribution. The results for the labor cost example using 1,000 iterations and 15 intervals to plot the histogram are shown in Figure 15. The mean cost value for the distribution is $249,408 and the standard deviation $47,527.

The power of this technique is even more apparent when the number of variables increases and the functional relationship becomes more complex. The application of this technique to a more complex example using an economic analysis is presented in the following chapter.

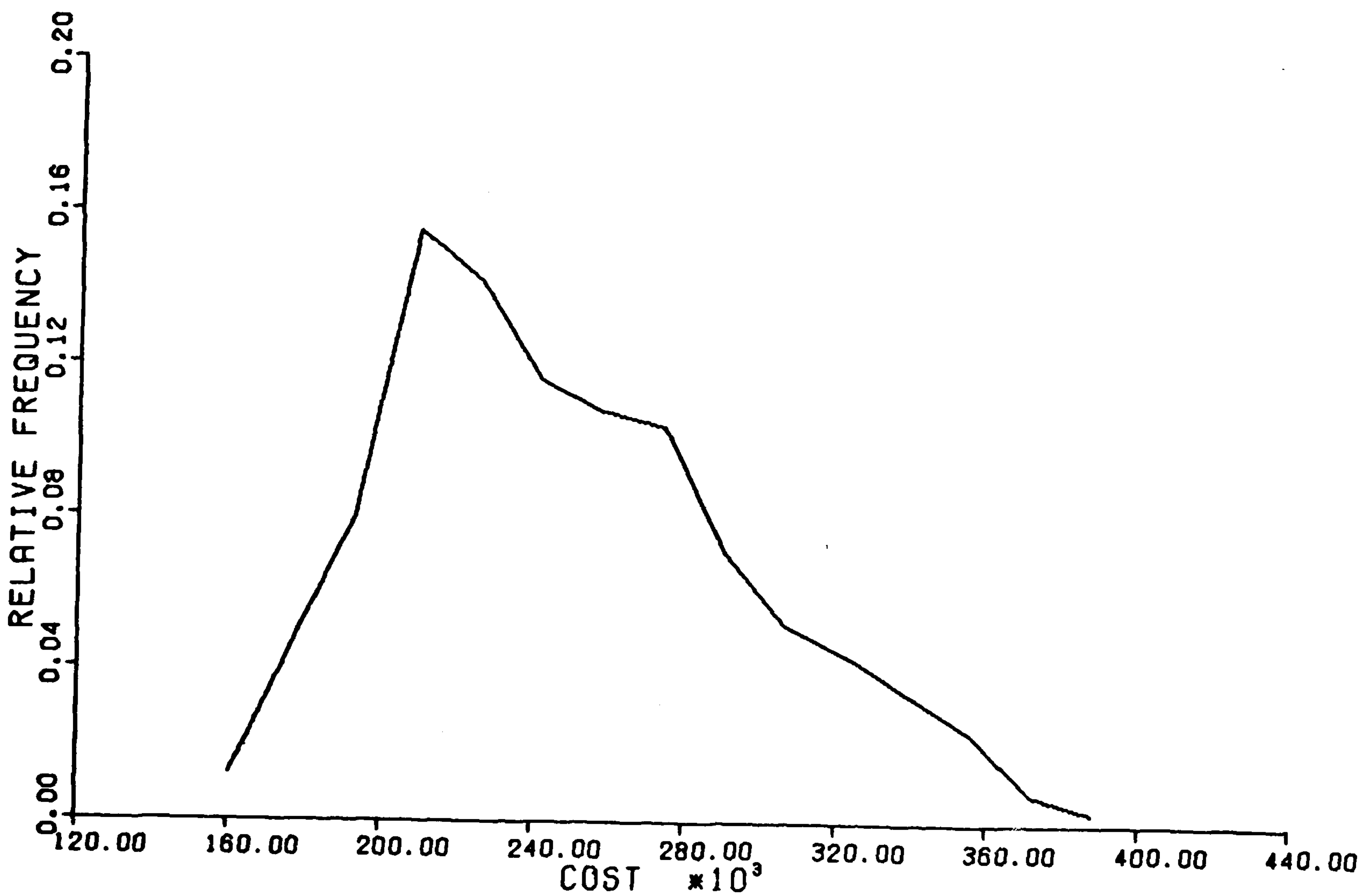

Fig 15. Example of Distribution for Labor Cost

V. An Illustrative Example of the Probabilistic Approach

In order to more adequately demonstrate the probabilistic approach described in Chapter 4, the writer felt that an example should be presented from a real world situation. Accordingly several economic analyses from project proposals within the Air Force Logistics Command were examined for possible inclusion as an illustrative example. Two criteria were used in choosing an appropriate example. First, it had to be complex enough to properly demonstrate the power of the simulation technique but no so complex that the analysis would be too large to deal with within the time constraints of this thesis effort. Second, in order to show the effects of different discount rates, the example had to cover a period of time which was long enough to make these effects apparent.

Using these criteria, an economic analysis for a military construction project was chosen which proposed the building of a new facility to be used to overhaul aeronautical hydraulic components. The only alternative to the new facility was the continued use of the existing facility. The analysis compared the relavant investment and operating costs for the two alternatives over a twenty-five year period.

The format ot this sample economic analysis presented a relevant cost comparison for the two alternatives on an item by item basis with the annual cost savings computed for each item. These cost savings were then discounted over the life of the project to determine the present value of the savings. A cost-benefit ratio was then computed by dividing the present value of the savings by the investment cost of the proposed project. A cost-benefit ratio greater than one would indicate that the project is economically feasible. This procedure is equivalent to the net present value criteria (present value of the savings minus investment cost) in which case a value greater than zero would indicate economic feasibility.

For purposes of this example the format was changed by summing all of the relavant cost variables (including investment cost were applicable) to determine total cost for each alternative. It should be noted that this format change in no way changes the results of the analysis. If single valued estimates of costs are summed using this format and the total discounted cost for the new facility is subtracted from that for the existing facility, the value obtained will be exactly the same as the net present value computed using the previous format. The advantage of this new format, however, is that probability distributions can be assigned to each cost variable and a distribution for the discounted cost for

each alternative determined using the Monte Carlo simulation technique. It is then possible to compare the cost distributions for the two alternatives rather than single valued estimates.

An examination of both alternatives revealed thirty-two relevant cost variables for the existing facility and thirty-four for the new facility. For those interested in the details of the cost variables, they are defined in Appendix A along with the cost equations used in the analysis. These specifics are not critical to an understanding of the stochastic approach presented here; however, some of the assumptions used will be discussed in order to familiarize the reader with the major issues encountered by the writer while performing the analysis.

The first of these issues concerns the choice of appropriate probability distributions for the variables. Only single valued estimates for the cost variables were included in the sample analysis with very little background information on how they were derived. Consequently, it was very difficult to determine realistically what the probability distribution for each of these variables might be. In order to demonstrate the simulation approach, therefore, it was necessary to make assumptions concerning these distributions. As noted in the previous chapter, the characteristics of the

triangular distribution make it suitable to represent cost variables. Thus, this distribution was chosen to represent all of the variables in the analysis.

The next step was to determine the low, most likely, and high values in order to define the triangular distribution for each variable. The point estimates given in the sample analysis could be used as the most likely values, but the low and high values still had to be determined. In actual practice these values would have been determined by subjective estimates from experts on these cost variables. But since the writer did not have access to such expert opinion, values were assigned more or less arbitrarily. The use of this arbitrary approach makes any actual application of the results of this analysis inappropriate but does not detract from the illustrative purpose of the example. The single valued estimates for each variable and the values used for the high and low estimates are tabulated in Appendix B.

The next major issue encountered in the analysis was the question of correlation of a given cost variable between time periods. The sample analysis assumed that some of the costs remained constant over the entire twenty-five years of the project. This is one extreme position in this issue and assumes perfect correlation of the variables from time period to time period. The opposite extreme is one of com-

plete independence or zero correlation. This is the assumption typically made in a Monte Carlo simulation where a separate random variate would be determined for the variable in each time period.

It should be recognized that there is a middle ground between these two extreme assumptions, that is, the existence of some functional relationship which defines the interdependence of the variable from one time period to the next. This relationship might take the form $X_t = \alpha X_{t-1}$ where X_t is the value of the variable in the present time period, X_{t-1} is the value in the previous time period, and α is a proportionality constant. While recognizing that this type of relationship is probably closer to reality than either extreme position, the determination of a realistic value for the proportionality factor could only be accomplished through an extensive evaluation of historical data. This kind of evaluation is certainly deserving of further research; however, it was far beyond the scope of the present analysis. This being the case, a decision was made to use one of the two extreme positions for the purpose of illustration.

In making a choice between these two extremes it seemed reasonable to assume that there is in fact some relationship between time periods. Because of this, the perfect correlation alternative was viewed to be more realistic than com-

plete independence. Thus, the procedure used for a given Monte Carlo trial in this analysis was to determine a random variate for the variable in year one and then use the same value in years two through twenty-five. It should be noted, however, that not all variables were treated in this way. For some it was readily apparent that the complete independence assumption was appropriate. In that case a separate random variate was determined for each year. Those variables for which the perfect correlation assumption was used are preceeded by an asterisk in Appendix B.

The final issue to be discussed concerns the treatment of variables between alternatives. Some of the variables in the analysis are used in the cost equations for both the new and the existing facilities. In the sample analysis these variables were assumed to be the same for both alternatives with the cost savings being determined by a proportionality factor which represented the percentage reduction in existing facility cost which would be realized in the new facility. An example using one of the cost equations from Appendix A will better illustrate this.

One portion of personnel cost is determined for the existing facility by

$$\frac{XE(1)}{XE(2)} \times XE(3)$$

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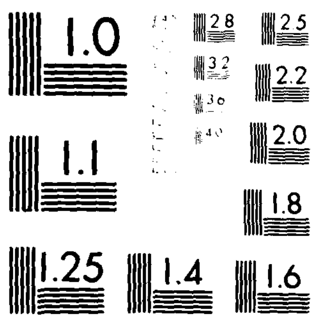

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS [illegible]

and for the new facility by

$$\frac{XN(1) \times [1 - XN(4)]}{XN(2)} \times XN(3)$$

where XE(1) and XN(1) both represent projected standard labor hours required, XE(2) and XN(2) represent the production efficiency factor, XE(3) and XN(3) represent labor cost, and XN(4) is the percentage work standard reduction in the new facility. In the sample analysis, each of the first three pairs of variables are equal. Thus, the cost savings realized was as a result of the work standard reduction only.

In the present analysis, each of these cost factors were treated as random variables in order to determine the cost distribution over many Monte Carlo trials. But to compare the cost of each alternative on any given trial one would like to know the effects of the work standard reduction, all other things being equal. This can be likened to a case in experimental design where an attempt is made to keep all factors between the test groups the same except the variable under consideration. To make the comparison valid in this analysis, therefore, all of the variables which are present in the cost equations for both alternatives were equated for each Monte Carlo trial. This fact comes into play when a

comparison is made between the alternatives as will be seen in the results below.

Using the assumptions described above, the Monte Carlo simulation was performed using 1,000 iterations or Monte Carlo trials, and a relative frequency histogram for total discounted cost was plotted using fifteen intervals. Discount rates of five, ten, and fifteen percent were used to demonstrate the differences which occur as a result of varying this rate. The simulation program uses the Fortran language, but since it was written specifically for this analysis and is not general in nature, it is not included in this paper. A copy is on file, however, and interested parties can receive further information by contacting Professor Joseph Cain, AFIT School of Engineering, Wright-Patterson Air Force Base, Ohio. The results of the analysis are presented below in both graphical and numerical form.

Results

Existing Facility Cost Data Using a 5 Percent Discount Rate

Single Valued Cost Estimate $140,349,063

Monte Carlo Data

Minimum Value	$140,134,342
Maximum Value	146,643,216
Mean Value	143,021,413
Standard Deviation	1,121,232
Coefficient of Variation	.0078
Interval Range	433,925

Midpoint of Interval	Relative Frequency
$140,351,304	.007
140,785,229	.021
141,219,154	.044
141,653,079	.084
142,087,004	.127
142,520,929	.130
142,954,854	.144
143,388,779	.136
143,822,704	.120
144,256,629	.082
144,690,554	.057
145,124,479	.029
145,558,404	.010
145,992,328	.006
146,426,253	.003

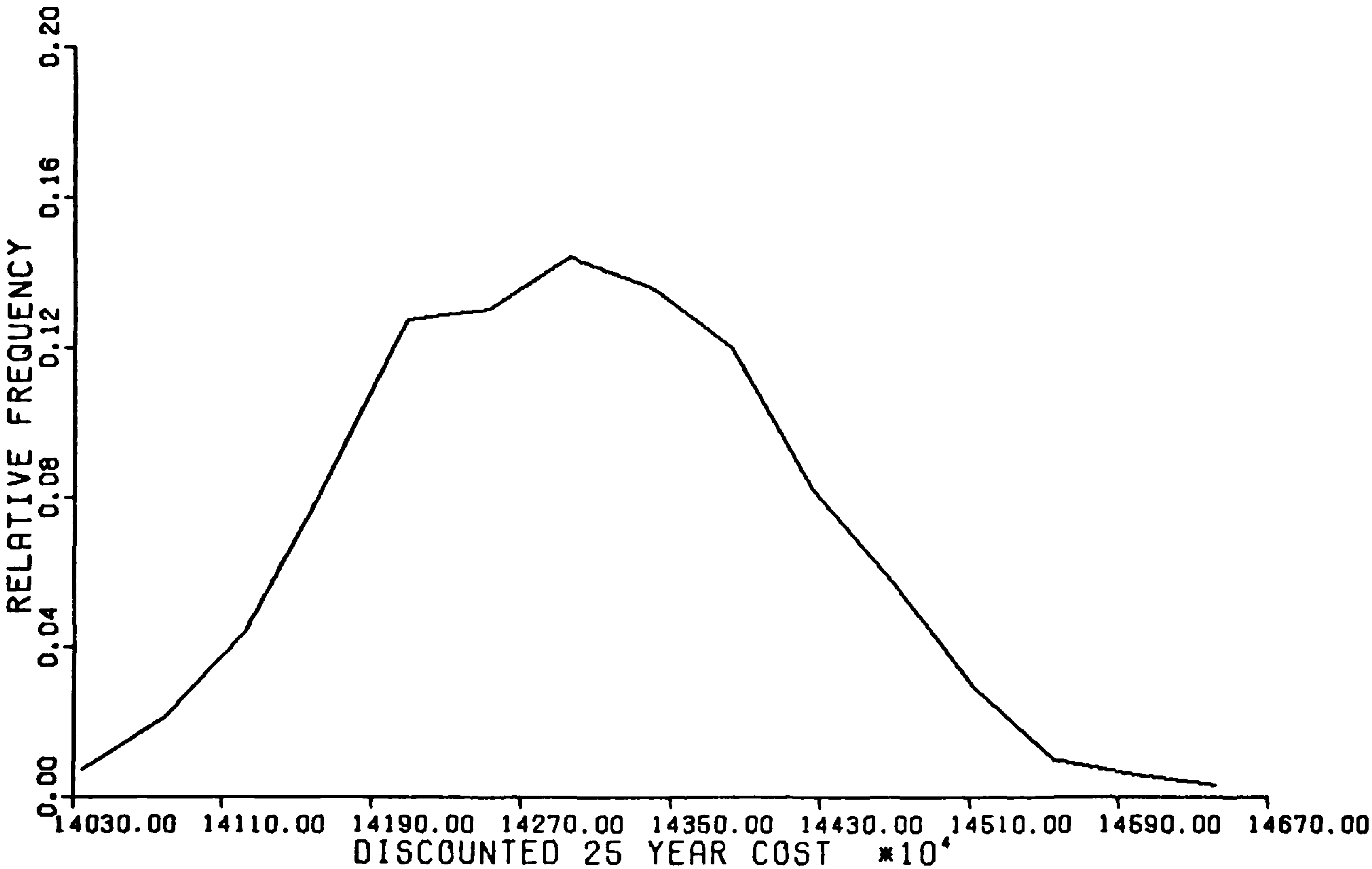

Fig 16. Cost Distribution for Existing Facility Using a 5 Percent Discount Rate

New Facility Cost Data Using a 5 Percent Discount Rate

Single Valued Cost Estimate $132,475,508

Monte Carlo Data

Minimum Value	$132,139,359
Maximum Value	139,141,047
Mean Value	135,477,791
Standard Deviation	1,159,969
Coefficient of Variation	.0086
Interval Range	466,779

Midpoint of Interval	Relative Frequency
$132,872,748	.004
132,839,527	.011
133,306,307	.023
133,773,086	.062
134,239,865	.106
134,706,644	.134
135,173,423	.146
135,640,203	.150
136,106,982	.128
136,573,761	.105
137,040,540	.070
137,507,320	.032
137,974,099	.017
138,440,878	.006
138,907,657	.006

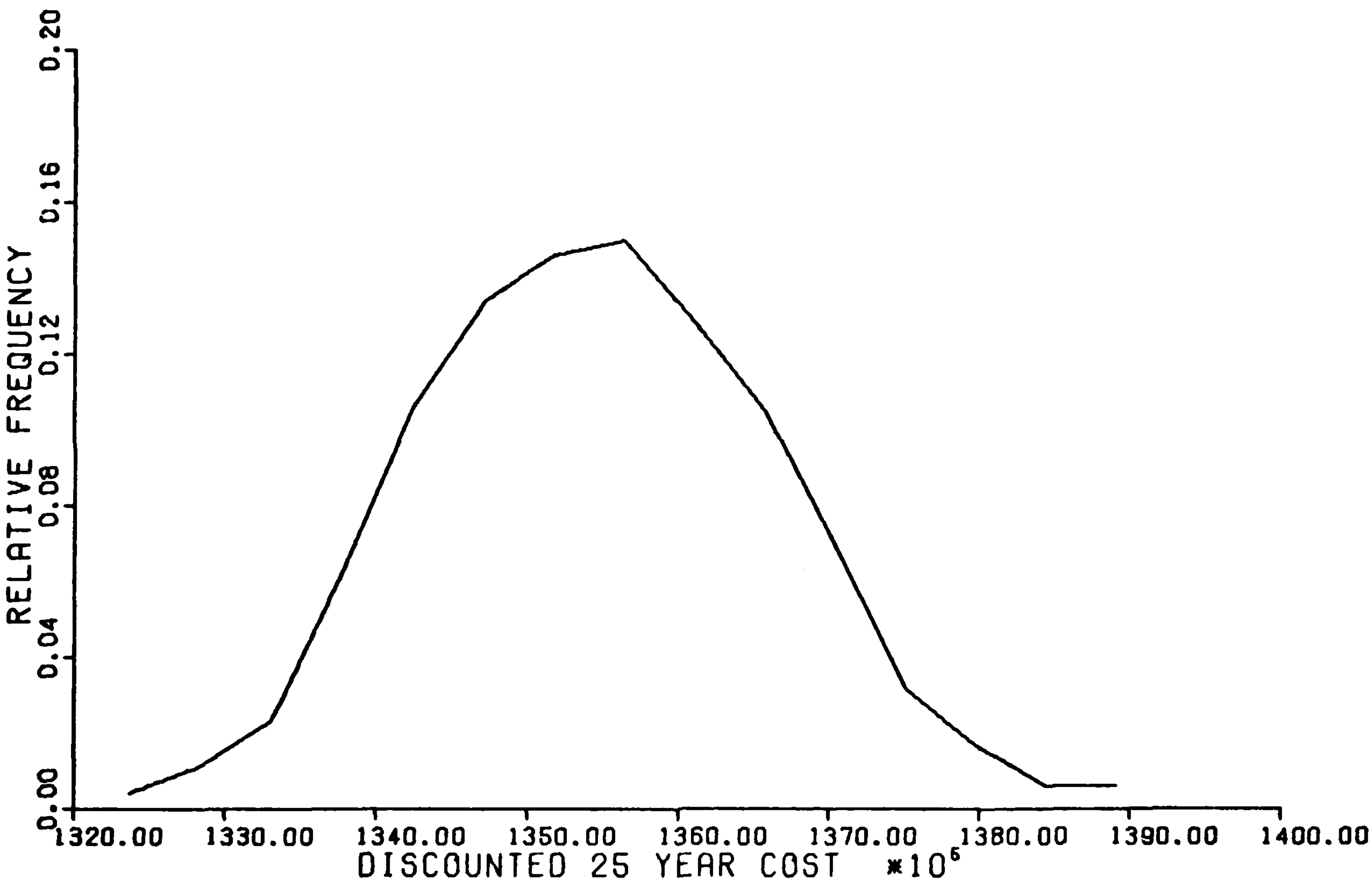

Fig 17. Cost Distribution for New Facility Using a 5 Percent Discount Rate

Existing Facility Cost Data Using a 10 Percent Discount Rate

Single Valued Cost Estimate $93,760,385

Monte Carlo Data

Minimum Value	$93,586,699
Maximum Value	98,012,401
Mean Value	95,531,692
Standard Deviation	770,576
Coefficient of Variation	.0081
Interval Range	295,047

Midpoint of Interval	Relative Frequency
$93,734,223	.011
94,029,269	.021
94,324,316	.046
94,619,363	.088
94,914,410	.117
95,209,547	.141
95,504,503	.149
95,799,550	.130
96,094,597	.120
96,389,644	.078
96,684,691	.046
96,979,737	.030
97,274,784	.016
97,569,831	.003
97,864,878	.004

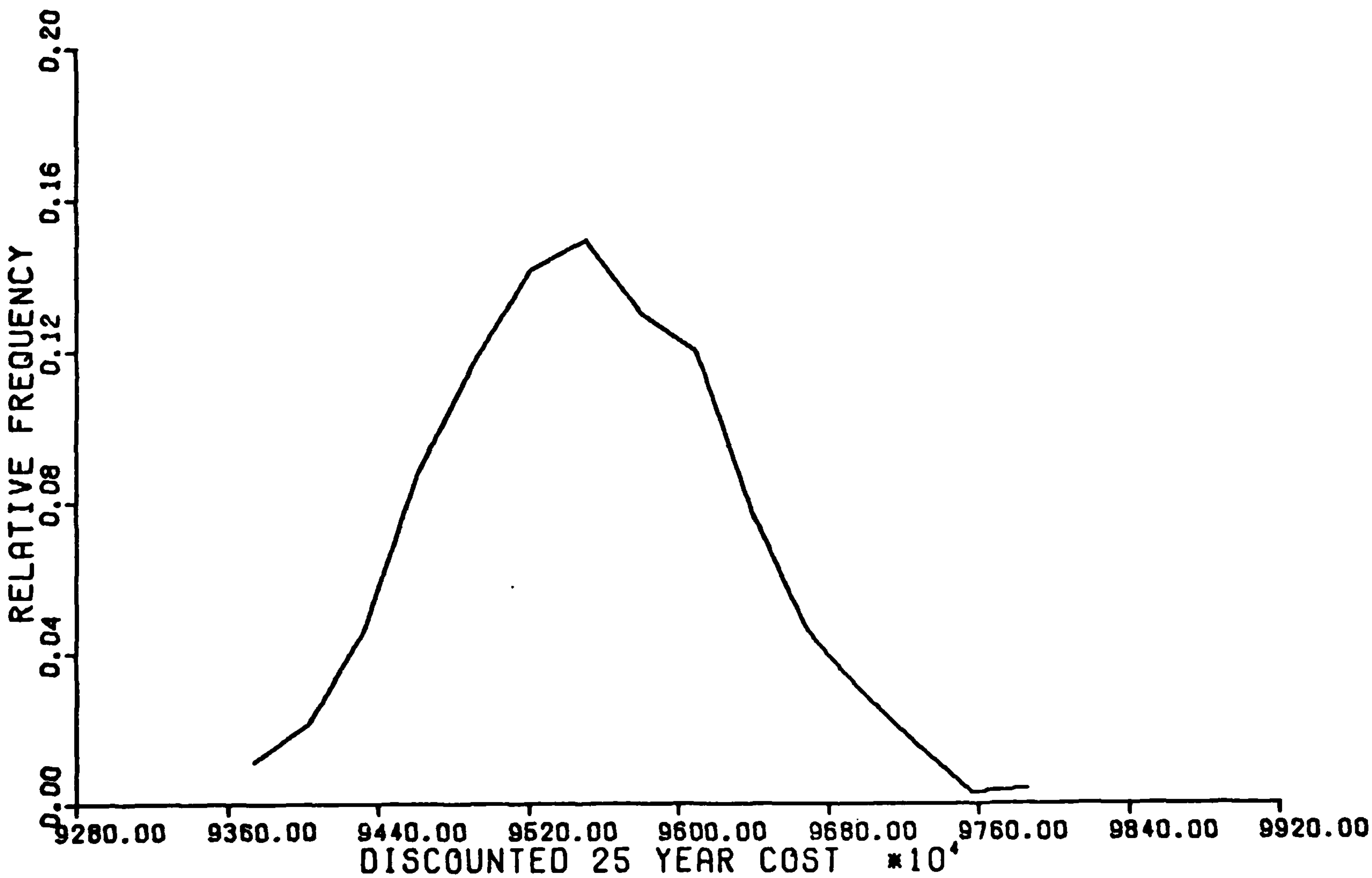

Fig 18. Cost Distribution for Existing Facility Using a 10 Percent Discount Rate

New Facility Cost Data Using a 10 Percent Discount Rate

Single Valued Cost Estimate $90,387,527

Monte Carlo Data

Minimum Value	$90,018,270
Maximum Value	94,862,776
Mean Value	92,423,842
Standard Deviation	809,243
Coefficient of Variation	.0088
Interval Range	322,967

Midpoint of Interval	Relative Frequency
$90,179,754	.003
90,502,721	.011
90,825,688	.015
91,148,655	.042
91,471,622	.102
91,794,589	.130
92,-17,556	.142
92,440,523	.153
92,763,490	.130
98,068,457	.108
93,409,424	.078
93,732,391	.050
94,055,358	.018
94,378,325	.010
94,701,292	.008

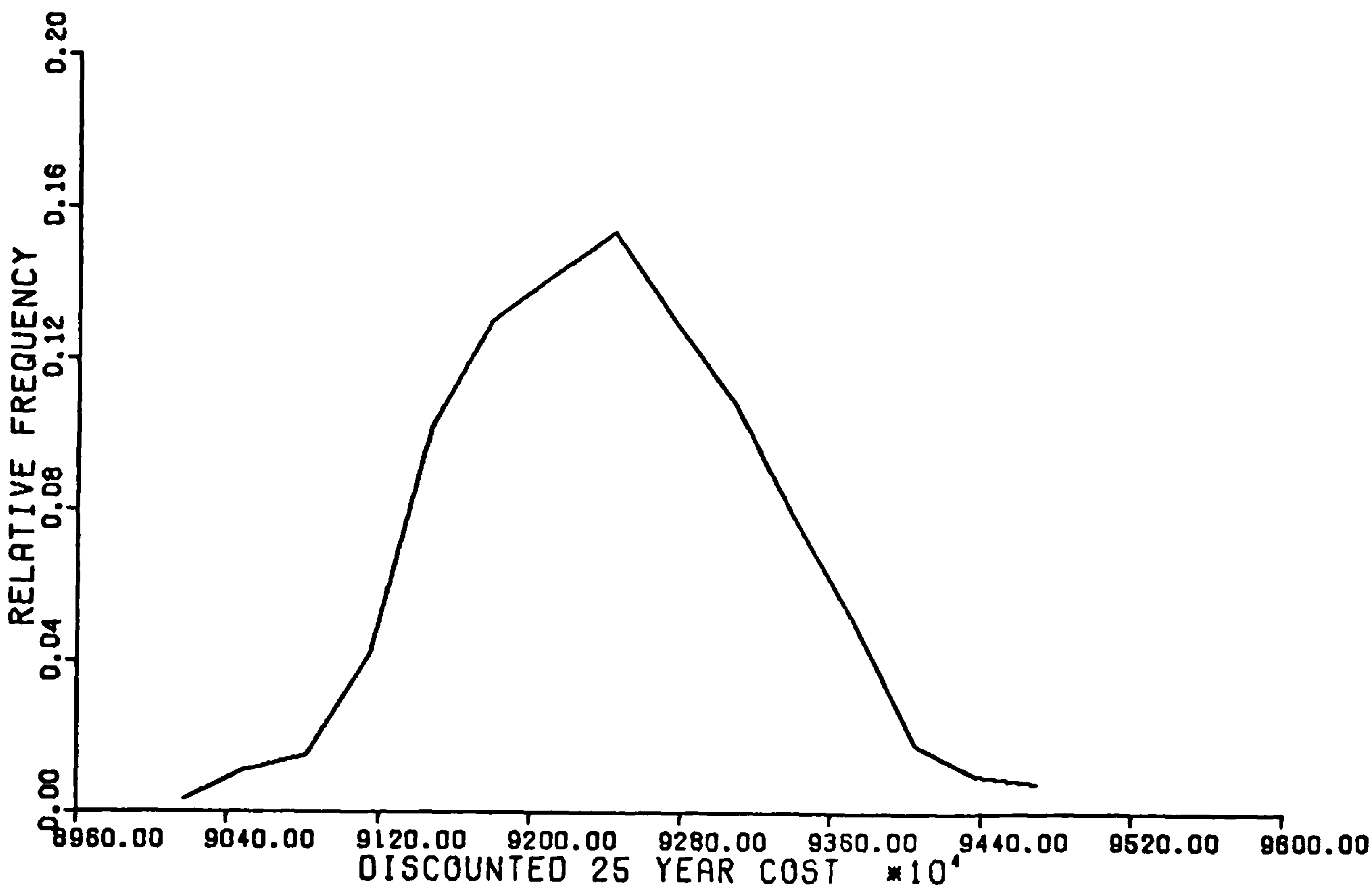

Fig 19. Cost Distribution for New Facility Using a 10 Percent Discount Rate

Existing Facility Cost Data Using a 15 Percent Discount Rate

Single Valued Cost Estimate $69,247,963

Monte Carlo Data

Minimum Value	$68,914,023
Maximum Value	72,420,679
Mean Value	70,547,747
Standard Deviation	592,586
Coefficient of Variation	.0084
Interval Range	233,777

Midpoint of Interval	Relative Frequency
$69,030,912	.004
69,264,689	.017
69,498,466	.028
69,732,243	.068
69,966,020	.109
70,199,797	.139
70,433,574	.150
70,667,351	.147
70,901,128	.127
71,134,905	.093
71,368,682	.056
71,602,459	.035
71,836,236	.014
72,070,013	.008
72,303,790	.005

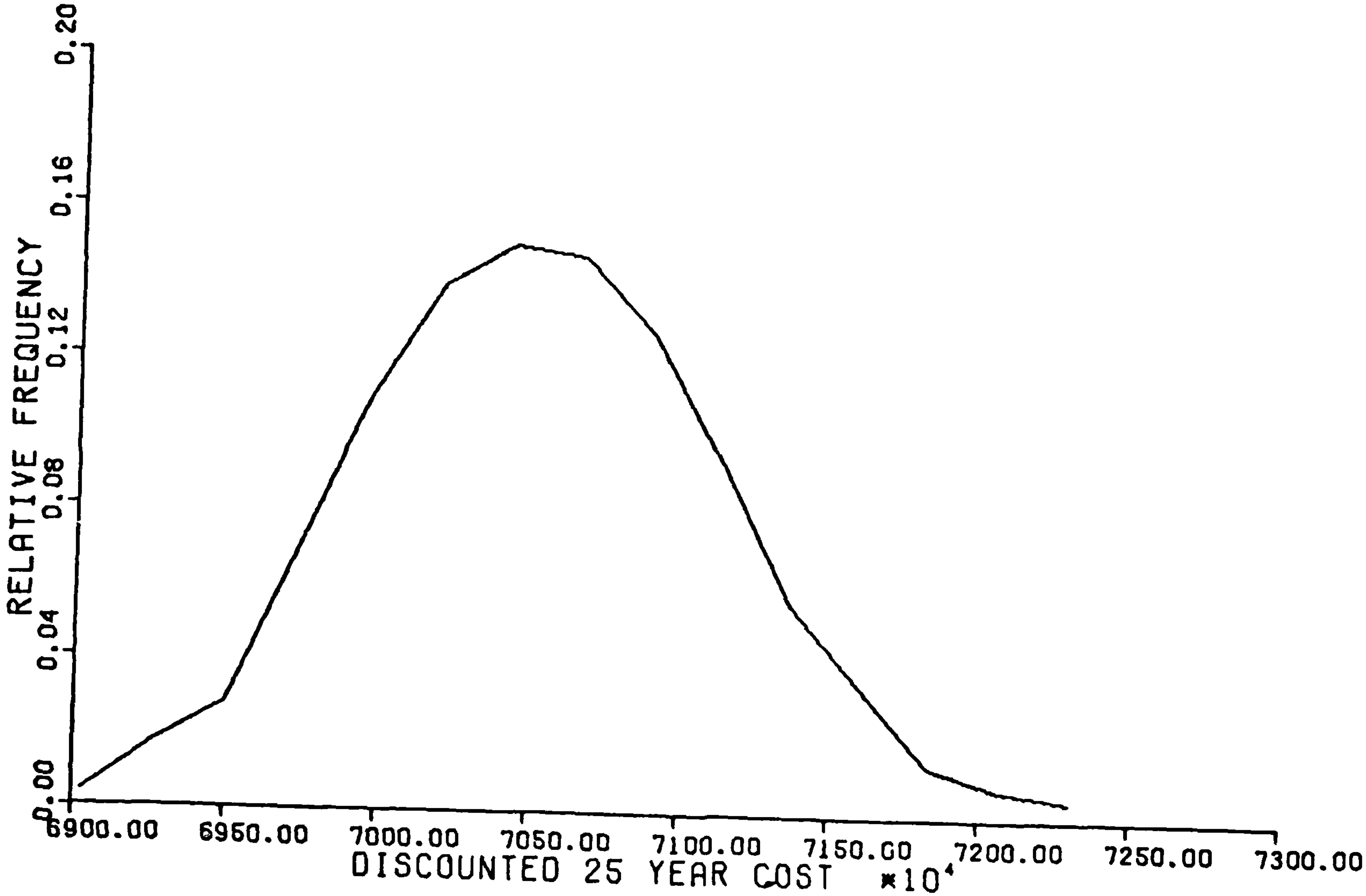

Fig 20. Cost Distribution for Existing Facility Using a 15 Percent Discount Rate

New Facility Cost Data Using a 15 Percent Discount Rate

Single Valued Cost Estimate $68,318,009

Monte Carlo Data

Minimum Value	$67,991,112
Maximum Value	71,762,438
Mean Value	69,851,757
Standard Deviation	634,977
Coefficient of Variation	.0091
Interval Range	251,422

Midpoint of Interval	Relative Frequency
$68.116,823	.003
68,368,244	.009
68,619,666	.025
68,871,088	.045
69,122,510	.099
69,373,931	.117
69,625,353	.152
69,876,775	.153
70,128,197	.137
70,379,618	.098
70,631,040	.079
70,882,462	.050
71,133,884	.018
71,385,305	.008
71,636,727	.007

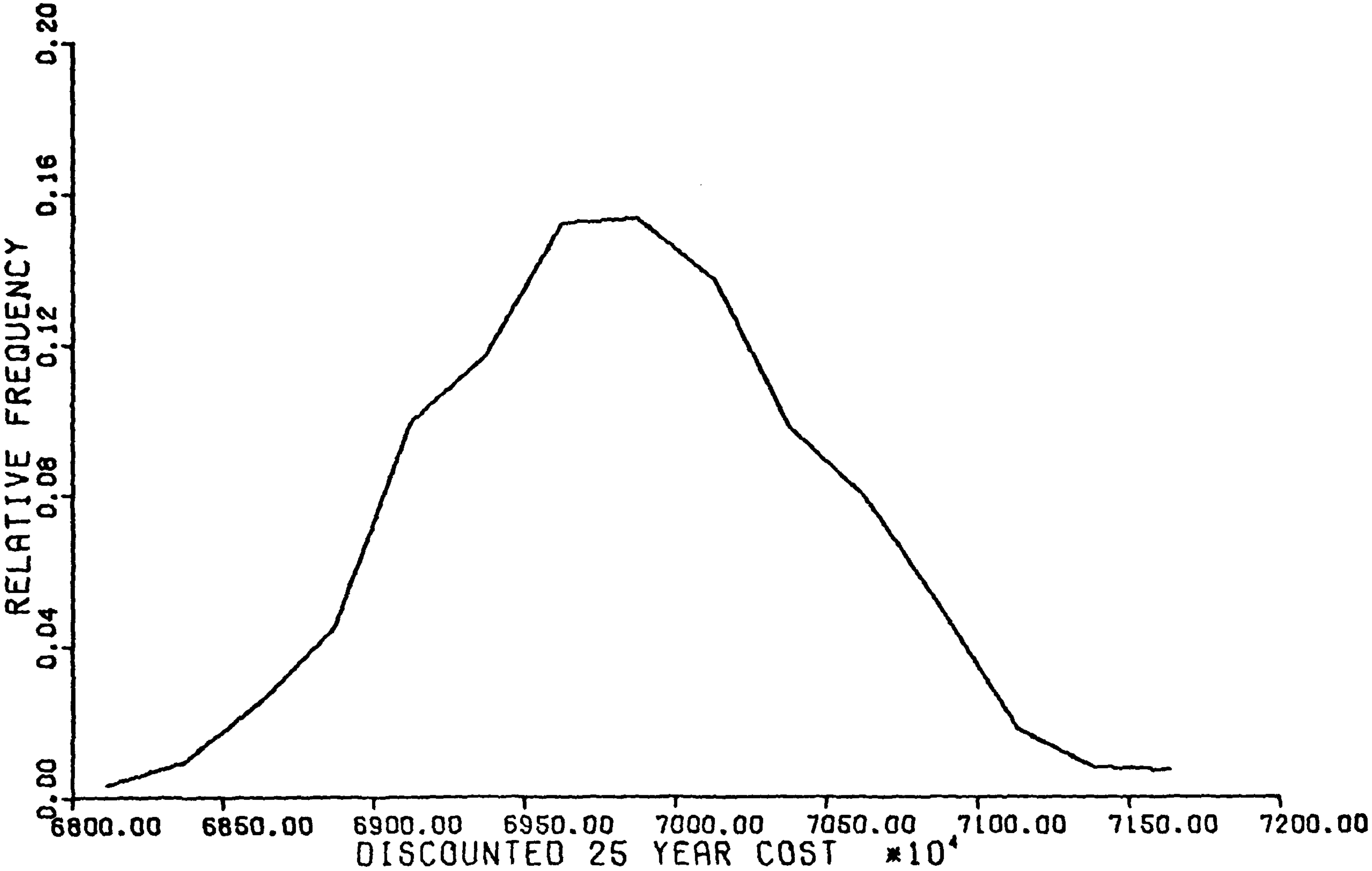

Fig 21. *Cost Distribution for New Facility Using a 15 Percent Discount Rate*

From the results of this analysis two observations can be made. First, in every case the cost computed using the single valued cost estimates are lower than the mean value for the probability distribution for both the existing and new facilities. Although this is probably due in part to the bias of the writer in the choice of the high and low values for the individual probability distributions, past experience has shown a higher tendency for underestimating than overestimating. This trend is evidenced by the numerous instances of "cost overruns" within the Department of Defense. This being the case, this kind of result is probably not unrealistic.

Second, the uncertainty of each alternative can be evaluated by observing the standard deviation and coefficient of variation for each distribution. In this case both are lower for the existing facility than for the new facility but the differences are not significant. Again, this is the kind of result that one would expect since the uncertainty involved in building and operating a new facility would be slightly greater than it would be for continued operation of the existing one.

In addition to these observations, it would also be desirable to compare the cost distributions for the two alternatives. An intuitive approach would be to plot both distributions together to see if there is any "overlap"

which would indicate the possibility of the cost of the new facility exceeding the cost of the existing one. If the two distributions were completely independent, this comparison would be valid. As noted earlier in the chapter, however, the two distributions are not independent due to the fact that some of the variables were equated between alternatives for the purpose of a valid comparison. The only way of comparing the distributions for total cost between alternatives, therefore, is to subtract the total discounted cost of the new facility from that for the existing facility on each Monte Carlo trial. This process results in "sample values" for net present savings which can be plotted in a relative frequency histogram just as was done for cost. This histogram would then be a valid comparison of the savings possible between alternatives.

Figures 22 through 24 show this distribution of savings for each discount rate. These figures also show the effect of increasing the discount rate with the distributions moving to the left (savings being less) as the discount rate is increased. The distributions determined for the five and ten percent rates show a zero probability of net present savings being negative since neither of the distributions lie in the "negative savings" range. The distribution determined using a fifteen percent rate, however, shows some probability that this could occur as indicated by the

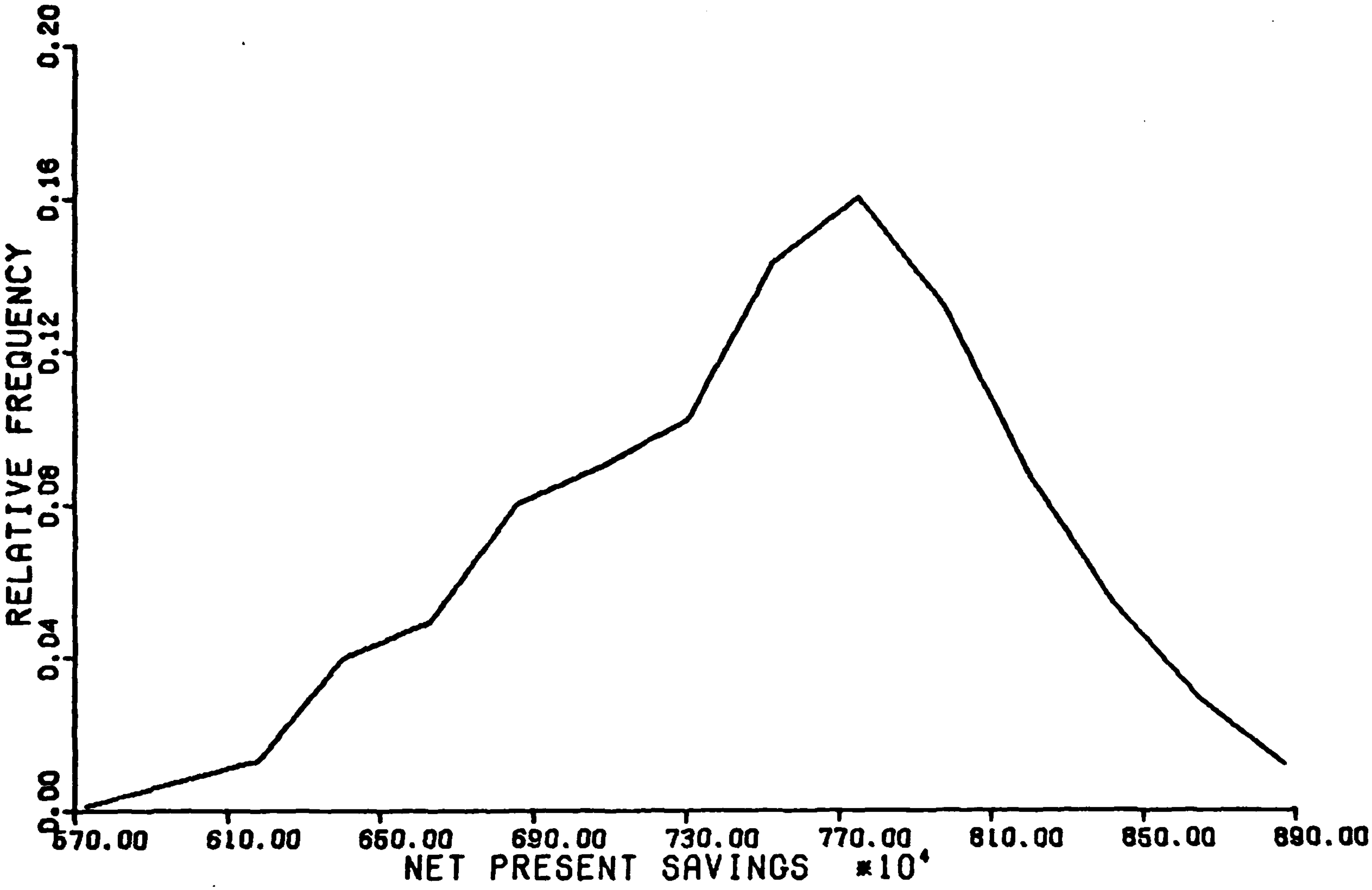

Fig 22. Savings Distributed Using a 5 Percent Discount Rate

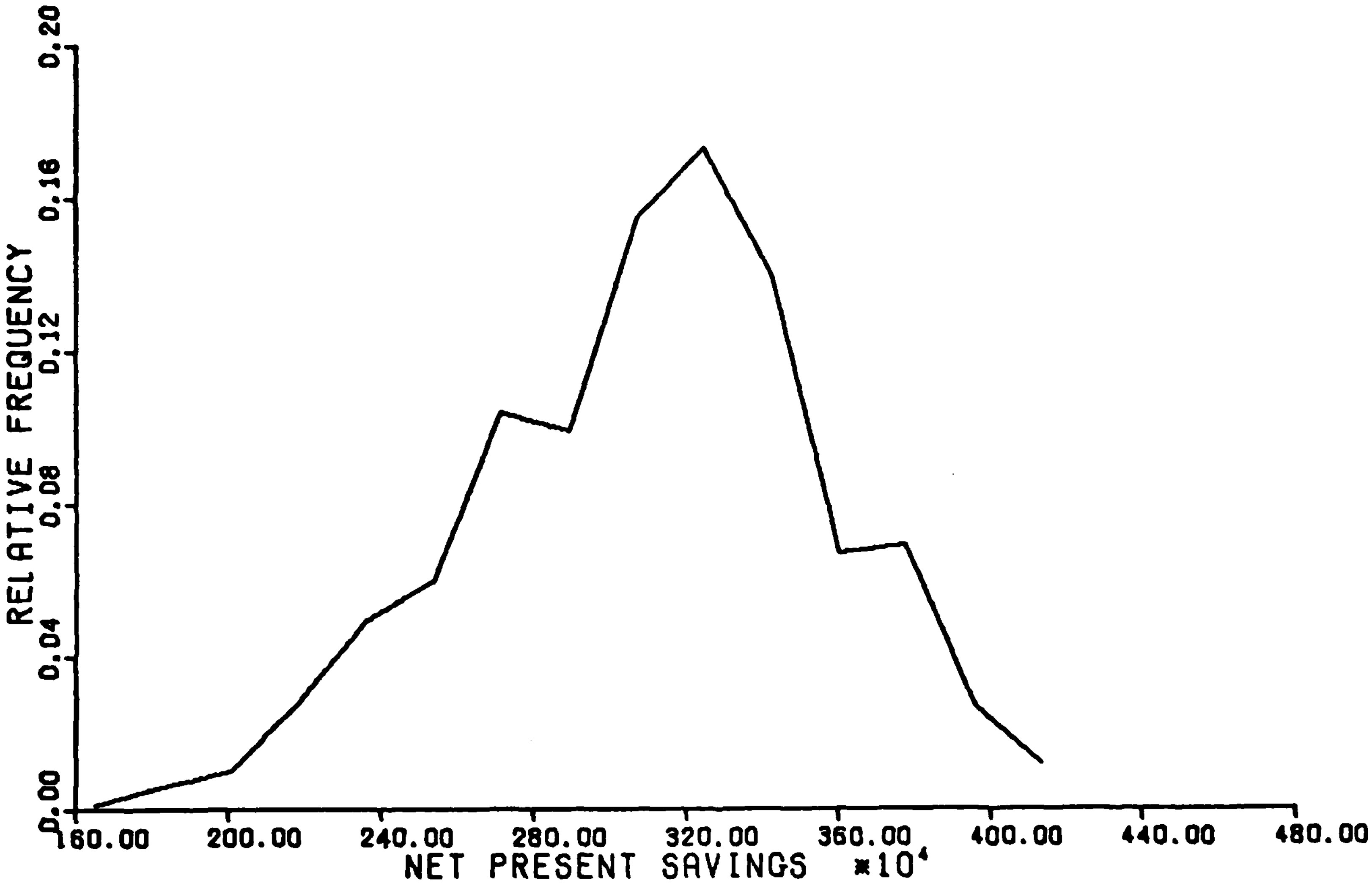

Fig 23. Savings Distribution Using a 10 Percent Discount Rate

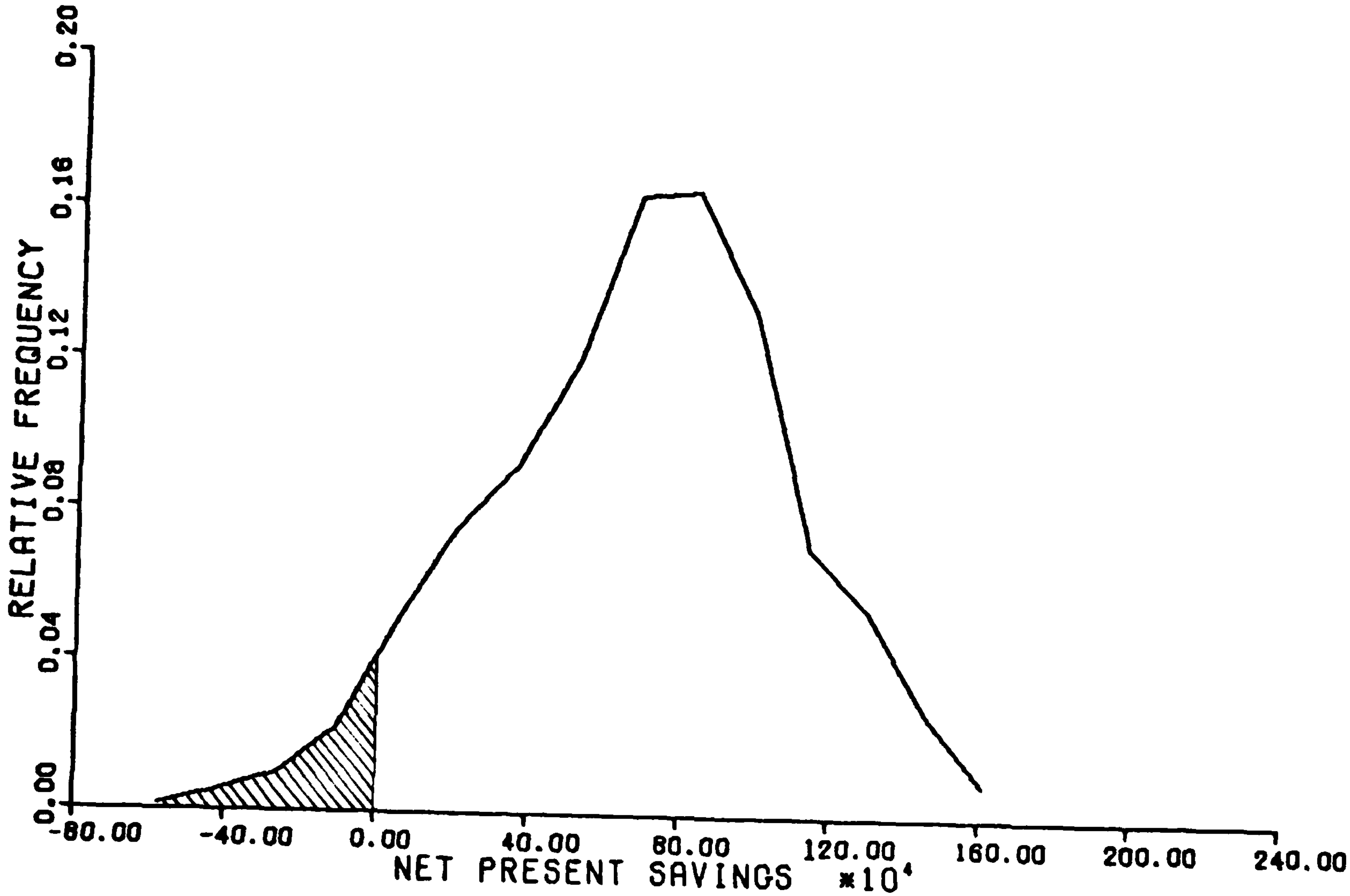

Fig 24. Savings Distribution Using a 15 Percent Discount Rate

hash marked area in Figure 24. This probability can be approximated by estimating that portion of the total area under the curve which lies in the hash marked area. For the example in Figure 24, this is between four and five percent which means that the probability that the new facility will show no net savings is between .04 and .05 assuming a fifteen percent discount rate.

In summary, the use of a probabilistic approach in economic analyses can provide valuable information to the decision maker. It provides a means of assessing the uncertainty present in individual alternatives by providing a probability distribution for cost. It also provides information about the shape or skewness of the distribution which could be very useful in making decisions on such things as the amount of contingency funds to be allotted to a project. In addition, this approach provides a means of comparing alternatives and making probabilistic statements concerning the likelihood of realizing cost savings.

VI. Summary, Conclusions, and Recommendations

Summary and Conclusions

This study investigated the issues in the debate over the choice of a social discount rate for use in evaluating public investments in general and DOD investments in particular. Two major controversies were examined in detail. The first arises over a dispute as to whether the social discount rate should reflect social objectives or opportunity cost. It was shown that those who favor a rate which reflects social objectives rely on a social time preference function for its determination. Since these social objectives are only accomplished through government action, they contend that the marketplace will not reflect the proper rate to be used in discounting these social investments. Those who advocate the opportunity cost principle, on the other hand, believe that direct observation of interest rates (or rates of return) in the marketplace is the only valid way of determining individual time preferences. Thus, these market rates represent an opportunity cost if funds are diverted from private to public investments. Evaluation of public investments at the opportunity cost rate would insure that society is not less wealthy than it would have been if the funds had been left in the private sector.

The social time preference viewpoint was rejected in this study, and by most economists, for both practical and theoretical reasons. On the practical side, there is no way to objectively determine what society's time preference function really is. This leaves the choice of the discount rate to subjective estimate which would then make it subject to manipulation for purposes other than serving society's best interest. The major theoretical shortcoming is the fact that historical data on economic growth indicate that each generation has been wealthier than that which preceeded it. If this trend continues there is no theoretical basis for increasing government investment for the sake of future generations since they will be wealthier than the one at present.

A review of the literature concerning the social opportunity cost discount rate revealed that there are also unresolved issues even among those who accept its validity. The major argument is over which interest rates observed in the private sector should be used to represent the opportunity cost. The rates used depend on which private investments are assumed to be displaced by undertaking public projects. Even though there is no way to determine which assumptions are correct, empirical calculations of discount rates using various assumptions all result in values in the

seven to ten percent range. Thus, the ten percent DOD rate seems to adequately reflect the opportunity cost of investment funds.

The second major issue examined was the question of whether risk compensation in investments is a social as well as a private cost. The view that it is not a social cost infers that a riskless rate should be used as the social discount rate. The main argument in favor of this viewpoint is that government spreads the risk of its investments over a large number of people and thus, the risk to each individual is near zero. The key assumption is that the average covariance among the rates of return on these investments is zero. It has been shown, however, that this assumption does not hold. Thus, the riskless rate is also rejected.

The opposing view holds that proper representation of opportunity cost must include the risk premiums that are included in the returns on private investment. Some confusion on this point was evident in the literature in that some believe that making explicit allowance for risk in an investment analysis would make the use of a riskless rate appropriate. This view ignores the opportunity cost issue, however. It was concluded that, even if explicit allowance is made for risk, the discount rate used must still include the risk premiums on private investment returns as an opportunity cost. Even unique public projects, such as many of

those undertaken in DOD, must include an allowance for the risk premiums present in the private sector. Allowances for the inherent risk of an investment is a separate issue from an allowance for the opportunity cost. The writer concluded that allowances for opportunity cost should be incorporated in the discount rate while consideration of inherent risk should be accomplished through the use of probability distributions for the cost and benefit variables.

The remainder of the study presents a methodology which can be used to evaluate inherent risk through the use of probability distributions. Numerous methods for generating probability distributions through subjective estimates and objective data were discussed along with some of the most commonly used theoretical distributions. A discussion of the use of Monte Carlo simulation to generate a probability distribution for the cost of an investment was then presented. The study concludes with an application of this methodology to an example based on an actual economic analysis which had been performed within Air Force Logistics Command.

Recommendations for Further Study

Although a study of the major issues surrounding the debate over the social discount rate is informative and leads to a better understanding of the controversy, it does not seem likely that further study on this topic will resolve

any of these issues. The differences of opinion which occur are primarily based on the assumptions used. For the most part it is impossible to determine conclusively which assumptions are correct. Consequently, a consensus of opinion as to "the" appropriate rate is a remote possibility regardless of how much research is conducted.

Further research should prove to be far more fruitful in the area of stochastic models for investment decision. Acceptance of probabilistic information for making investment decisions is largely a matter of convincing decision makers that this information adequately captures the uncertainty present in the investment project. To do this, a more rigorous methodology must be developed. This methodology must include means for determining the uncertainty present in individual variables through subjective and empirical data. It must also include techniques for determining the functional relationships which exist between variables in order to adequately simulate the possible outcomes of the investment. This last issue was touched on in Chapter 5 when discussing the relationship between the values of cost variables in different years.

This study is not the first to explore these issues and hopefully it will not be the last since a great deal of work remains to be done. Further study in this area could greatly enhance the validity of a probabilistic approach.

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Appendix A

Monte Carlo Cost Variables and Cost Equations

The following is a listing of the relevant cost variables and cost equations used in the Monte Carlo simulation of the economic analysis example presented in Chapter 5. Variables in the cost equations for the existing facility are labeled XE(I) and those for the new facility XN(I). Some of the variables appear in both the existing and new facility equations. When this situation exists, the two variables are both listed to show the equivalence. The total cost of each facility is broken down into several component costs for easier understanding.

<u>Personnel Cost</u>

XE(1),XN(1)	Projected standard labor hours required
XE(2),XN(2)	Production efficiency factor
XE(3),XN(3)	Labor cost per hour
XN(4)	Percentage work standard reduction in new facility
XE(4)	Engineering and planning personnel travel hours between existing work areas
XE(5)	Engineering and planning personnel cost per hour

Personnel cost for the existing facility is then computed as follows:

$$\frac{XE(1)}{XE(2)} \times XE(3) + XE(4) \times XE(5)$$

The following formula is used to compute the same cost for the new facility:

$$\frac{XN(1) \times [1 - XN(4)]}{XN(2)} \times XN(3)$$

The cost difference occurs due to work standard reduction and the elimination of the travel time for engineering and planning personnel.

<u>Equipment Preventive Maintenance and Repair Cost</u>

XE(6)	Preventive maintenance and repair cost of equipment that could be released under new facility alternative
XE(7)	Preventive maintenance and repair cost for each hydraulic test stand
XN(5)	Percent of existing facility cost for each test stand that would be incurred in the new facility

The cost component for the existing facility is determined by

$$XE(6) + XE(7) \times 30$$

and for the new facility by

$$XN(5) \times XE(7) \times 30$$

The difference in this cost component occurs due to a new system which would eliminate part of the equipment required in the existing facility and reduce the operating cost for the hydraulic test stands.

Utilities and Services Cost for the Building

All costs are per 1,000 square feet of building area.

XE(8),XN(6)	Fuel oil cost
XE(9),XN(7)	Natural gas cost
XE(10),XN(8)	Electricity cost
XE(11),XN(9)	Water cost
XE(12),XN(10)	Sewage and industrial waste disposal cost
XE(13),XN(11)	Heat and steam cost
XE(14),XN(12)	Air conditioning cost
XE(15),XN(13)	Electrical repair and maintenance cost
XE(16),XN(14)	Refuse collection cost
XE(17),XN(15)	Custodial supply cost
XN(16)	Percent of existing facility cost for fuel oil, gas, and air conditioning (per 1,000 square feet) which will be required in the new facility
XN(17)	Percent of existing facility cost of electricity (per 1,000 square feet) which will be required in the new facility

Cost for utilities and services for the existing facility is computed as follows:

$$[XE(8) + XE(9) + XE(10) + XE(11) + XE(12) + XE(13) + XE(14) + XE(15) + XE(16) + XE(17)] \times 80.04$$

The same cost component for the new building is computed as follows:

$$[XN(6) \times XN(16) + XN(7) \times XN(16) + XN(8) \times XN(17) + XN(9) + XN(10) + XN(11) + XN(12) \times XN(16) + XN(13) + XN(14) + XN(15)] \times 100$$

Although the new facility cost for this component would be lower per thousand square feet due to more efficient use of utilities, the total utilities cost would be higher because a larger building is used.

Utilities Cost for Equipment

XE(18),XN(18)	Hours per year of equipment operation
XE(19),XN(19)	Electricity cost per megawatt hour

Cost for the existing facility

$$XE(18) \times XE(19) \times .755325$$

Cost for the new facility

$$XN(18) \times XN(19) \times .07833$$

The cost difference occurs due to a change in the constant term which reflects the use of a central hydraulic system to run test stands rather than running them with electric motors.

Cost of Maintenance and Repair of Facility

All costs per 1,000 square feet of building area.

XE(20),XN(20)	Maintenance and repair cost of maintenance and production space
XE(21),XN(21)	Maintenance and repair cost of material processing and holding space
XE(22),XN(22)	Maintenance and repair cost of administrative/logistical space
XN(23)	Percent of existing facility cost (per 1,000 square feet) that will be incurred in the new facility

The maintenance and repair cost for the existing facility is computed by

$$69.884 \times XE(20) + 2.651 \times XE(21) + 7.05 \times XE(22)$$

where the constants are the areas (in thousands of square feet) used for each function. The maintenance and repair cost for the new facility is similarly computed as follows:

$$[68.51 \times XN(20) + 16 \times XN(21) + 15.49 \times XN(22)] \times XN(23)$$

The cost difference occurs because of lower maintenance and repair cost for a new facility.

Opportunity Cost

If the new facility is built, part of the present facility will be used to house a laboratory. If the new facility is not built, a building must be constructed to house the

lab. Thus, the construction cost of the lab was used as an opportunity cost of maintaining the hydraulic facility in its existing building.

XE(23)	Construction cost of primary lab facility
XE(24)	Construction cost of the lab support facility
XE(25)	Construction supervision and administration cost
XE(26)	Design cost
XE(27)	Construction cost of converting the existing facility to use as a lab
XE(28)	Supervision and administration cost of the conversion
XE(29)	Design cost for the conversion

The opportunity cost for the existing facility alternative is the computed as follows:

XE(23) + XE(24) + XE(25) + XE(26) - XE(27) - XE(28) - XE(29)

Production Equipment Investment Cost

The required production equipment differs between alternatives but the useful life in both cases was assumed to be ten years. Equipment investment costs are incurred in years one, ten, and twenty with only half of the actual cost in year twenty being used due to the assumed twenty-five year life of the overall project. The equipment cost is then represented by the following variables:

Existing Facility

XE(30)   Equipment cost in year 1

XE(31)   Equipment cost in year 10

XE(32)   Equipment cost in year 20

New Facility

XN(24)   Equipment cost in year 1

XN(25)   Equipment cost in year 10

XN(26)   Equipment cost in year 20

New Facility Investment Cost

XN(27)   Construction cost for the primary facility

XN(28)   Construction cost for the support facility

XN(29)   Construction supervision and administration cost

XN(30)   Initial outfitting equipment (I.O.E.) cost

XN(31)   Supervision and administration of I.O.E.

XN(32)   Design cost

XN(33)   Shop relocation cost

XN(34)   Value of part of present facility for alternative use (over and above that which was listed under Opportunity Cost)

The total investment cost is then computed by summing XN(27) through XN(33) and subtracting XN(34).

For purposes of this analysis, all of the investment costs were assumed to be incurred in year zero for discounting purposes. This is not totally in conformance with the treat-

ment in the economic analysis, but the assumption does not change the results significantly nor detract from the illustrative purpose of the example.

To compute total discounted cost, the relevant cost components are summed for each year and discounted to present value. It should be noted that what is called total cost in this example is actually total <u>relevant</u> cost since cost components which would be the same for both alternatives are not included in the analysis.

Appendix B

Distribution Parameters for Cost Variables

In the tables below are presented the low, most likely, and high cost estimates which were used to define the triangular distributions for the cost variables in the Monte Carlo example in Chapter 5. The tables also show the relevant project years for each variable. As noted in Chapter 5 and Appendix A, some of the cost variables are the same for both alternatives. Consequently, only those variables which are unique to the new facility are listed in Table III.

Table II

Parameters for Existing Facility Cost Variables

Variable	Low Value	Most Likely Value	High Value	Relevant Years
XE(1)	500,000	526,109	560,000	1-25
XE(2)	.9	.95	.97	1-25
*XE(3)	15.00	15.16	15.50	1-25
XE(4)	400	500	525	1-25
*XE(5)	15.70	15.80	16.00	1-25
XE(6)	15,300	17,539	20,400	1-25
XE(7)	900	934	968	1-25
*XE(8)	275	283.50	350	1-25
*XE(9)	350	373.30	400	1-25
*XE(10)	275	290.40	350	1-25
*XE(11)	65	68.80	73	1-25
*XE(12)	180	189.40	195	1-25
*XE(13)	200	214.40	250	1-25
*XE(14)	40	45.50	55	1-25
*XE(15)	33	38.60	42	1-25
*XE(16)	65	74.10	80	1-25
*XE(17)	18	20.80	22	1-25
XE(18)	1975	2080	2215	1-25
*XE(19)	18.85	19.87	24.00	1-25
XE(20)	375	427.56	475	1-25
XE(21)	25	50.19	60	1-25
XE(22)	400	621	1,800	1-25
XE(23)	3,830,000	4,034,473	4,640,000	1
XE(25)	250,000	264,724	305,000	1
XE(26)	316,620	333,552	383,640	1
XE(27)	677,000	713,000	820,000	1
XE(28)	33,850	36,000	41,000	1
XE(29)	42,000	42,780	51,660	1
XE(30)	7,750,000	7,795,068	7,900,000	1
XE(31)	7,750,000	7,795,068	7,900,000	10
XE(32)	3,875,000	3,897,534	3,950,000	20

Table III

Parameters for New Facility Cost Variables

Variable	Low Value	Most Likely Value	High Value	Relevant Years
*XN(4)	.075	.09	.095	1-25
*XN(5)	.38	.4	.5	1-25
*XN(16)	.83	.85	.87	1-25
*XN(17)	.985	.99	1.0	1-25
*XN(23)	.79	.8	.85	1-25
XN(24)	7,162,000	7,204,119	7,300,000	1
XN(25)	7,162,000	7,204,119	7,300,000	10
XN(26)	3,581,000	3,602,059.50	3,650,000	20
XN(27)	6,020,000	6,336,110	7,285,000	0
XN(28)	1,333,000	1,402,890	1,615,000	0
XN(29)	367,650	387,000	445,000	0
XN(30)	580,000	583,000	590,000	0
XN(31)	29,000	29,150	29,500	0
XN(32)	500,000	527,369	607,000	0
XN(33)	470,000	475,000	500,000	0
XN(34)	300,000	336,090	350,000	0

Vita

The author received a Bachelor of Science degree in Mechanical Engineering from Michigan State University in 1969 and entered the Air Force in 1970. Following pilot training, he remained in Air Training Command as a T-38 instructor pilot until 1975. He then served as a B-52 Aircraft Commander until August 1978. In July 1978 he completed requirements for a Master of Science degree in Business Administration from the University of Northern Colorado through an on base education program. He was assigned to the Air Force Institute of Technology in August 1978 to pursue a master's degree in Systems Management.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this study the writer examines the debate over the choice of an appropriate discount rate for evaluating public investments. The conclusions drawn from the study are then applied to investment decisions within the Department of Defense. Two major issues are examined in detail. In the first, the social time preference discount rate is rejected, for both practical and theoretical reasons, in favor of an opportunity cost of capital rate which considers the alternative use in the private sector of the funds

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used in public investment projects. The second point of controversy centers on the use of a riskless discount rate versus a rate that includes the risk compensation for private investments. The riskless discount rate is rejected because it does not adequately portray the opportunity cost of public investment funds.

While a rate that includes the risk compensation present in the private sector adequately portrays the opportunity cost, it does not consider the risk inherent in the project. The writer concludes that consideration of inherent risk is a separate issue from the choice of a discount rate. One possible approach for consideration of inherent risk is presented through the use of probability distributions for cost and benefit variables in the investment analysis and the use of Monte Carlo simulation to generate a probability distribution for the net present value of an investment. The study concludes with an application of this probabilistic approach to an economic analysis which had previously been accomplished within the Air Force Logistics Command.

8